PROBLEMS IN DESCRIPTIVE GEOMETRY

BARTLETT

REVISED EDITION

NUMERICAL PROBLEMS

IN

DESCRIPTIVE GEOMETRY

FOR

CLASS AND DRAWING ROOM PRACTISE

REVISED EDITION

By

G. M. BARTLETT, M. A.

Instructor in Descriptive Geometry and Drawing in the University of Michigan

Published by the Author
Ann Arbor, Michigan
1907

The Ann Arbor Press, Printers

CLASSIFICATION OF LINES AND SURFACES

- **Lines**
  - Right lines
  - Plane curves (circle, ellipse, parabola, hyperbola, spirals, roulettes, etc).
  - Space curves (helix, spherical epicycloid, loxodrome, etc.)
- **Surfaces**
  - **Generated by Right Lines**
    - **Planes**
    - **Single Curved Surfaces** (developable)
      - All positions of generatrix parallel (cylinders)
      - All positions of generatrix intersect in common pt. (cones)
      - Consecutive positions of generatrix intersect two and two; no three positions intersecting in common pt. May be generated by a system of tangents to a line of double curvature (helical convolute.)
    - **Warped Surfaces** No two consecutive positions of generatrix in same plane; hence, non-developable.
      - Plane directer and two linear directrices.
        - Two rt. lines (hyper. parab.)
        - One rt. line and one curve (conoid)
        - Two curves (cylindroid)
      - Three linear directrices
        - Three rt. lines (hyperboloid of one nappe)
        - One curve, two rt. lines
        - Two curves, one rt. line (corne de vache)
        - Three curves
      - Curves generated by a rt. line touching a space curve and another rt. line, and making a constant angle with a fixed plane (helicoid)
  - **Double Curved Surfaces**
    - **Surfaces of Revolution** (sphere, ellipsoid of rev., torus, parab. of rev., etc.)
    - **Surfaces** of double curvature not generated by a revolving line nor by a rt. line. (ellipsoid).

ABBREVIATIONS

alt. = altitude.
bet. = between.
const. = construct.
cyl. = cylinder.
dia. = diameter.
dist. = distance or distant.
proj. = projection.
pt. = point.
rt. = right.
tang. = tangent.
G. L. = ground line.
H = horizontal or horizontal plane.
P = profile or profile plane.
V = vertical or vertical plane.
$\perp$ = perpendicular.
$\|$ = parallel.
$\angle$ = angle.
$\angle^s$ = angles.

ELEMENTARY PROPOSITIONS

1. The distance of a point from the H plane is the distance of its V projection from the ground line. The distance of a point from the V plane is the distance of its H projection from the ground line.

2. If any two projections of a point are given, the point is fully determined, and the third projection may be found from the above principle.

3. If a line is parallel to either plane of projection, its projection upon the other plane of projection is parallel to the ground line.

4. If a line is parallel to either plane of projection, it will be projected upon that plane in its true length.

5. If two intersecting lines are each parallel to the same plane of projection, the angle between the lines will be projected upon that plane in its true magnitude.

6. If a point lies in a given line, its projections will lie in the corresponding projections of the line.

7. If two lines are parallel in space, their corresponding projections are parallel.

8. The H and V traces of a plane always intersect the ground line at the same point.

9. If any two traces of a plane are given, the plane is fully determined.

10. If a plane is parallel to the ground line, its traces are parallel to the ground line, and conversely.

11. If a plane is perpendicular to either plane of projection, its trace upon the other plane is perpendicular to the ground line, and conversely.

12. If a line lies in a given plane, its H piercing point lies in the H trace of the plane, and its V piercing point lies in the V trace of the plane.

13. If a line is perpendicular to a plane, its projections will be perpendicular to the corresponding traces of the plane, and conversely.

14. If a line lies in a plane and is parallel to H (or V), its H (V) projection is parallel to the H (V) trace of the plane, and its V (H) projection is parallel to the ground line.

15. If two planes are parallel in space, their corresponding traces will be parallel, and conversely.

16. If a point in space be rotated about a line either lying in or parallel to H, the H projection of the point will always lie in the same perpendicular to the H projection of the line.

17. If a point in space be revolved into H about a line lying in H, its position in H will be at a distance from the axis equal to the hypotenuse of a right triangle whose legs are respectively the distance from the H projection of the point to the H projection of the line, and the distance from the V projection of the point to the ground line.

18. If a line not parallel to H be revolved about an axis intersecting it and parallel to H, into a position where both are parallel to H, any point in the revolved line will be horizontally projected at a distance from the H projection of the axis equal to the hypotenuse of a right triangle whose base is the distance of the H projection of the point (before revolution) from the H projection of the axis, and whose altitude is the distance of the V projection of the point (before revolution) from the V projection of the axis.

PROPOSITIONS RELATING TO WARPED SURFACES

1. The rectilinear elements of an hyperbolic paraboloid divide the directrices proportionally, and *conversely*.

2. If two right lines be divided into any number of proportional parts, the right lines joining the correspond-

ing points of division will line in a system of parallel planes, and hence be elements of an hyperbolic paraboloid, the plane directer of which is parallel to any two of these lines.

3. If any two rectilinear elements of an hyperbolic paraboloid be taken as directrices, with a plane directer parallel to the first directrices, and a surface be thus generated, it will be identical with the first surface.

4. Thru any point of an hyperbolic paraboloid, two rectilinear elements can always be drawn.

5. If two warped surfaces having two directrices have a common plane directer, a common rectilinear element and two common tangent planes, the points of contact being on the common element, they will be tangent all along this element.

6. If two warped surfaces have an element in common and are tangent to each other at three points of the same, they are tangent along the entire element.

REPRESENTATION OF POINTS, LINES AND PLANES

1. Show the projs. of the following points properly lettered and with distances given.

The pt. A, 1″ behind V, 1½″ below H.
The pt. B, lying in V, 1″ below H.
The pt. C, 3″ in front of V, 1″ above H.
The pt. D, 1″ behind V, lying in H.
The pt. E, 2″ behind V, 1½″ below H.
The pt. F, 1″ in front of V, 1″ below H.
The pt. G, lying in V, 2″ above H.
The pt. J, 1″ in front of V, lying in H.
The pt. K, lying in V, lying in H.
The pt. L, in 3rd quadrant, 1″ from H, 2″ from V.
The pt. M, in 2nd quadrant, 3″ from H, 2″ from V.
The pt. N, in 1st quadrant, 1½″ from H, 3½″ from V.
The pt. P, in 4th quadrant, 4″ from H, 1″ from V.

2. State in what quadrant each of the points shown in the figure are located, and whether the point is nearer H or V.

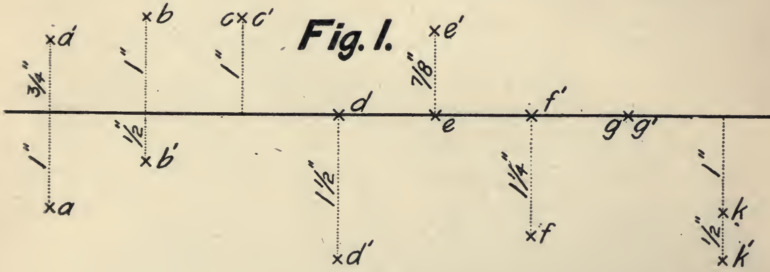


Fig. I.

3. Show the projs. of the lines

AB, || to H, || to V, in 3rd quadrant.

CD, || to H, perpendicular to V, in 2nd qaudrant.

EF, || to H, inclined to V, in 1st quadrant.

GH, inclined to H, || to V, in 1st quadrant.

JK, ⊥ to H, parallel to V, in 2nd quadrant.

MN, inclined to H, inclined to V, in 1st quadrant.

OP, inclined to both planes of proj. and in a plane perpendicular to the G. L., in 1st quadrant.

QR, inclined to V, and lying in H. beyond G. L.

ST, inclined to H, and lying in V above G. L.

UV, lying in both H and V.

4. Const. the projs. of two lines, AB and AC, intersecting in A, one || to H, the other || to V.

5. Show the projs. of a line joining a point A in the 2nd quadrant with a point B in the 3rd quadrant.

6. Show the projs. of a line joining a point C in the 4th quadrant with a point D in the 1st quadrant.

7. Describe the situation of the following lines with respect to the *ground line,* the *planes of projection* and the *quadrants.*

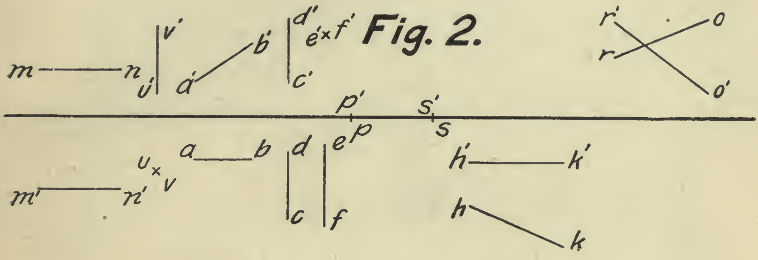


Fig. 2.

8. If the H proj. of a line is || to the G. L., what conclusion do you draw (*a*) as to the *position* of the line? (*b*) as to its intersection with the V plane? (*c*) as to the V trace of any plane passed through the line?

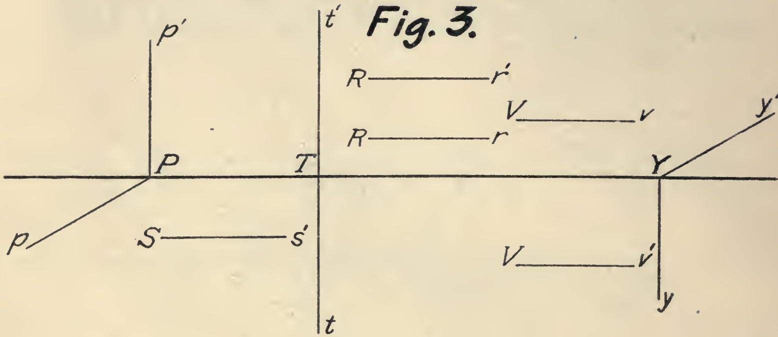
9. If a right line lies in a given plane, what conclusion do you draw as to the points where the line pierces the H and V planes respectively?

10. Represent by its traces the planes:

aAa′, ⊥ to both H and V.
bBb′, inclined to H; ⊥ to V.
cCc′, ⊥ to H; inclined to V.
dDd′, inclined to both H and V.
eEe′, ⊥ to H; || to V.
fFf′, || to H; ⊥ to V.
gGg′, || to G. L., but not passing thru it.
hHh′, containing the G. L.

11. Describe the situation of the following planes with respect to the *ground line,* the *planes of projection,* and the *quadrants.*

Fig. 3.



12. If the H trace of a plane is ⊥ to the G. L., what conclusion do you draw (*a*) as to the *position* of the plane? (*b*) as to the ∠ between the plane and the H plane? (*c*) as to the V proj. of any line lying in the plane? (*d*) as to the ∠ bet. the plane and the V plane?

PROBLEMS RELATING TO THE POINT, LINE AND PLANE

13. Find the H and V piercing pts. of the line MD, Fig. 4.

14. Find the H and V piercing pts. of the line SB, Fig. 4. Const. *double size.*

15. Find the H and V piercing pts. of the line DN, Fig. 5. Const. double size.

16. Find the H and V piercing pts. of the line BM, Fig. 6.

17. Find the H and V piercing pts. of the line CD, Fig. 4.

18. Find the H and V piercing pts. of the line BM, Fig. 4.

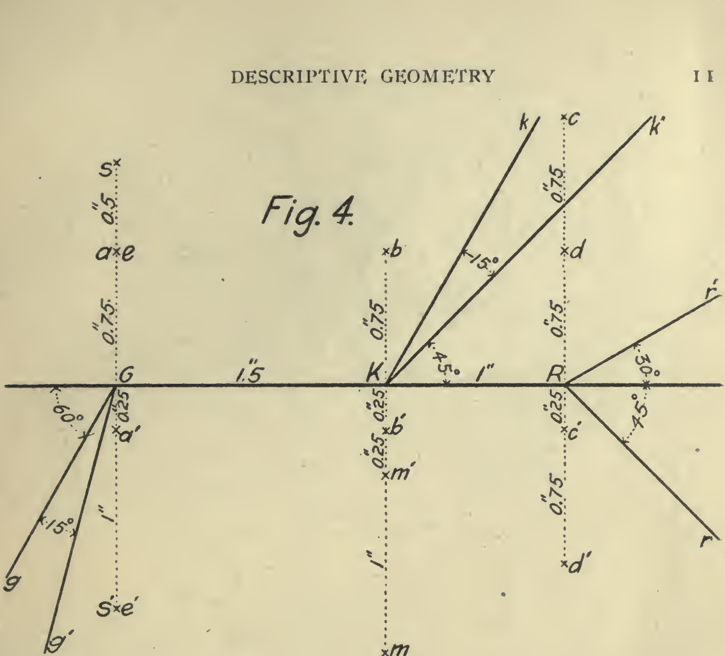


Fig. 4.

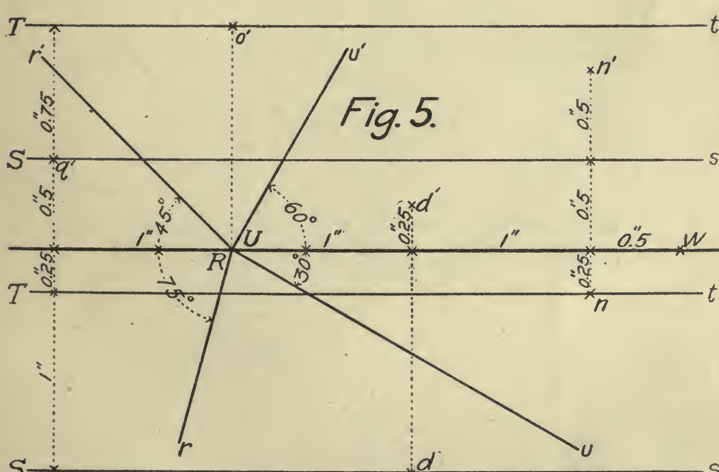
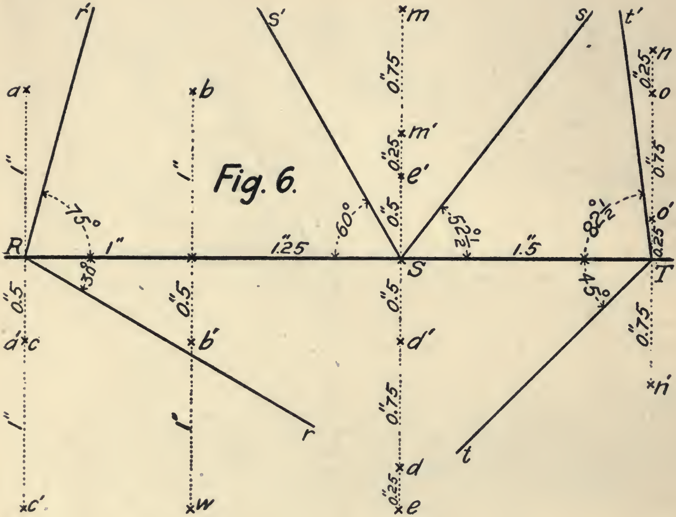
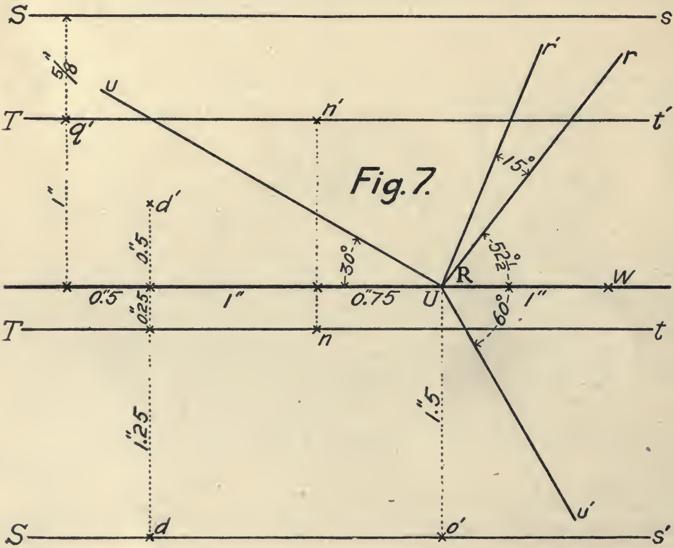


Fig. 5.


r'
s'
m
s
t'
n
a
b
0.75
0.25
o
m'
1"
1"
0.25
e'
0.75
75°
60°
0.5
52½°
82½°
o'
R
1"
1.25
1.5
0.25
38°
S
T
45°
0.5
0.5
0.5
d' c
b'
d'
0.75
n'
1"
1"
0.75
r
t
d
0.25
c'
w
e
Fig. 6.


S
s
5/8"
r'
r
u
T
n'
t'
q'
15°
1"
d'
0.5
30°
52½°
R
W
0.5
0.25
1"
0.75
U
1"
60°
T
n
t
1.25
1.5
u'
S
d
o'
s'
Fig. 7.

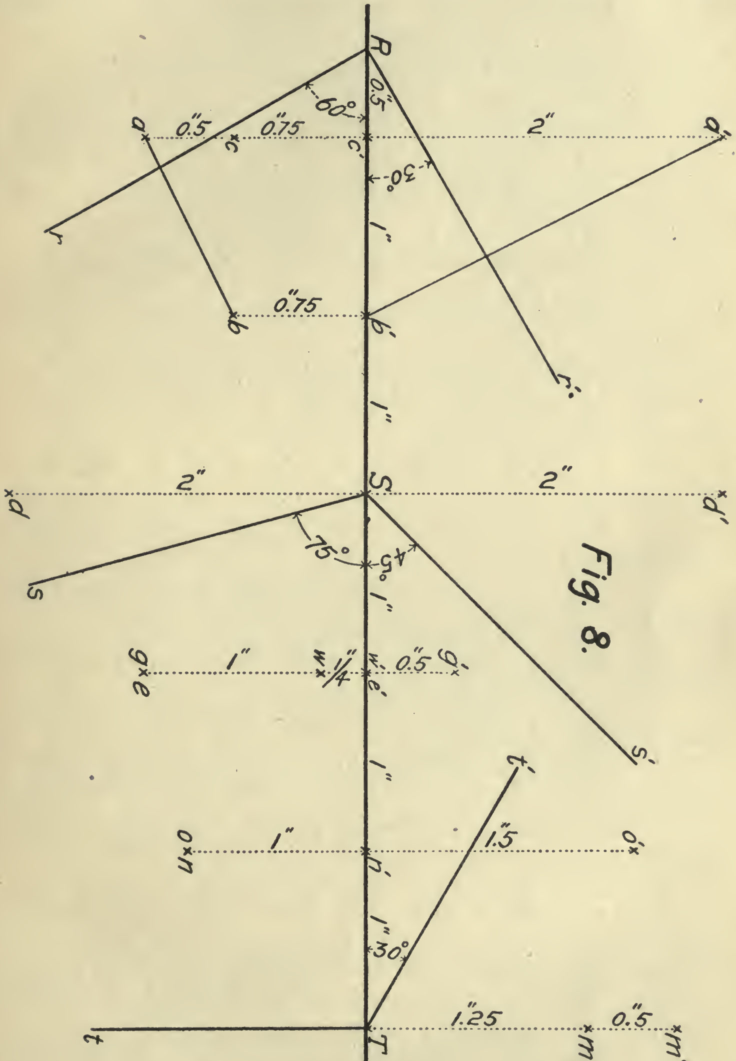


Fig. 8.

19. Find the H and V piercing pts. of the line MD, Fig. 6. Const. double size.

20. Find the H and V piercing pts. of the line AC, Fig. 6.

21. Const. the projs. of the line that pierces H at *b* and V at *m*′, Fig. 6.

22. Const. the projs. of the line that pierces H at *w* and V at *d*′, Fig. 6.

23. Const. the projs. of the line that pierces H at *c* and V at *m*′, Fig. 4.

24. Const. the projs. of the line that pierces H at *a* and V at *m*′, Fig. 4.

25. Find the length of the line EC, Fig. 4.

26. Find the length of SB, Fig. 4.

27. Find the length of MD, Fig. 4, by two methods.

28. Find the length of EN, Fig. 6, by two methods.

29. Find the length of CM, Fig. 6, by two methods.

30. Find the length of DE, Fig. 6, by two methods. Const. double size.

31. Find a point N on the line EC, Fig. 4, that is 2″ from E. Give the dist. of *n* from *e*.

32. Find a point O on the line MD, Fig. 4, 2″ from M. Give the dist. of *o* from *m*.

33. Find a point Q on the line EN, Fig. 6, 1″ from N. Give the dist. of *q* from *n*.

34. Find the ∠ which MD, Fig. 4 ,makes with H and with V.

35. Find the ∠ which EC, Fig. 4, makes with H, and with V.

36. Find the ∠ which EN, Fig. 6, makes with H, and with V.

37. The H proj. of a line is *dn*, Fig. 5. Its H trace is at *d*. The line makes an ∠ of 30° with H. Const. its V proj. Give dist. of *n*′ from G. L.

38. The H proj. of a line is *ce*, Fig. 6. Its H trace

is at *c*. The line makes an ∠ of $22\frac{1}{2}°$ with V. Const. its V proj. Give dist. of *c*′ from G. L.

39. Find the point A in the line DN, Fig. 5, equally dist. from H and V. Give dist. *da*. Const. double size.

40. Same as above, but equally dist. from H and P. Take the P plane thru N. Give dist. *da*.

41. Pass a plane, tTt′, thru A, M and D, Fig. 4. Give ∠ bet. traces on drawing.

42. Pass a plane, uUu′, thru B, C and E, Fig. 6. Give ∠ bet. traces on drawing.

43. Pass a plane, uUu′, thru B, D and M, Fig. 6. Give ∠ bet. traces on drawing.

44. Pass a plane, sSs′, thru the point C and the line AM, Fig. 4. Give ∠ bet. traces on drawing.

45. Pass a plane, tTt′, thru the point M and the line AB, Fig. 4. Give ∠ bet. traces on drawing.

46. Pass a plane, sSs′, thru the lines EC and MC, Fig. 4. Give ∠ bet. traces on drawing.

47. Pass a plane, uUu′, thru the lines BM and BE, Fig. 6. Give ∠ bet. traces on drawing.

48. Pass a plane, sSs′, thru the lines BC and DC, Fig. 4. Give ∠ bet. traces on drawing.

49. Pass a plane, kKk′, thru the lines CM and DM, Fig. 6. Give ∠ bet. traces on drawing.

50. Pass a plane, tTt′, thru MD and a line || to MD thru C, Fig. 4. Give ∠ bet. traces on drawing.

51. Pass a plane, kKk′, thru EN and a line || to EN thru B, Fig. 6. Give ∠ bet. traces on drawing.

52. Pass a plane, sSs′, thru the line AB and a line || to the G. L. thru M, Fig. 4. Give dist. bet. traces on drawing.

53. Pass a plane, kKk′, thru the line AB and a line || to the G. L. thru E, Fig. 6. Give dist. bet. traces on drawing.

54. Find the value of the ∠ EMD, Fig. 4.

55. Find the value of the ∠ SDM, Fig. 4.

56. Find the value of the ∠ BMN, Fig. 6. Draw double size.

57. Find the value of the ∠ BEN, Fig. 6. Draw double size.

58. Find the value of the ∠ ABS, Fig. 4. Draw double size.

59. Find the value of the ∠ ABM, Fig. 6. Draw double size.

60. Find the value of the ∠ ADN, Fig. 6.

61. Find the ∠ bet. the lines SE and SB, Fig. 4.

62. Find the ∠ bet. ON and OM, Fig. 8. Draw double size.

63. Find the ∠ in space bet. the traces of the plane rRr′, Fig. 4.

64. Find the ∠ in space bet. Uu and Uu′, Fig. 5.

65. Find the ∠ in space bet. the traces of the plane gGg′, Fig. 4.

66. Show the true size and shape of the triangle BCD, Fig. 4.

67. Show the true size and shape of the triangle BMN, Fig. 6. Draw double size.

68. Find the true size and shape of the triangle OGD, Fig. 8. Give lengths of the three sides.

69. Find the true size and shape of the triangle ADG, Fig. 8.

70. Find the bisector, CN, of the ∠ MCD, Fig 6. Give ∠ $m'c'n'$.

71. Find the bisector, DN, of ∠ SDM, Fig. 4. Give ∠ $s'd'n'$.

72. Find the bisector, EF of ∠ AEN, Fig. 6. Give ∠ $a'e'f'$. Draw double size.

73. Find the bisector, EF, of ∠ BEN, Fig. 6. Give ∠ $b'e'f'$. Const. *double size.*

74. Find the bisector, BN, of ∠ ABS, Fig. 4. Give ∠ $a'b'n'$. Const. double size.

75. Find the bisector, BN, of ∠ ABM, Fig. 6. Give ∠ *a′b′n′*. Const. double size.

76. Find the bisector, SN, of ∠ ESB, Fig. 4. Give ∠ *esn*.

77. Find the bisector, EN, of ∠ AEC, Fig. 4. Give ∠ *a′e′n′*.

78. Thru B, Fig. 4, pass a line BN, making an ∠ of 45° with EC. Give ∠ *bnc*. (Two solutions.)

79. Thru A. Fig. 4, pass a line, AN, ⊥ to SB. Give ∠ *anb*.

80. Find the pt. Q. where the line MD pierces the plane rRr′, Fig. 4. Give dist. bet. *q* and *q′* on drawing. Const. double size.

81. Find the pt. K where the line AB, Fig. 8, pierces the plane rRr′. Give dist. bet. *k* and *k′* on drawing.

82. Find the pt. Q where the line SM pierces the plane kKk′, Fig. 4. Give dist. bet. *q* and *q′* on drawing. Const. double size.

83. Find the pt. F where the line AB pierces the plane rRr′, Fig. 4. Give dist. BF.

84. Find the pt. F where a line thru D || to the G. L. pierces the plane uUu′, Fig. 5. Give dist. DF.

85. Find the pt. F, where the line AB pierces the plane kKk′, Fig. 4. Give dist. AF. Draw double size.

86. Find the pt. F, where the line DN pierces the plane sSs′, Fig. 5. Give dist. *fd*.

87. Find the pt. F, where the line DN pierces a plane ⊥ to the ground line and passing thru W, Fig. 5.

88. Find the pt. F, where the line CD pierces the plane kKk′, Fig. 4. Give dist. *fd*. Draw double size.

89. Find the pt. F, where the line AC pierces the plane rRr′, Fig. 8. Give dist. *f′a′*.

90. Find the pt. F, where the line BM pierces the plane rRr′, Fig. 4. Give dist. *fb*. Draw double size.

91. Find the pt. F, where EC pierces the plane of MD

and BD, Fig. 4, without constructing the traces of the plane. Give dist. *cf*. Draw double size.

92. Find the pt. F, where BE pierces the plane of MN and CN, Fig. 6, without constructing the traces of the plane. Give dist. *bf*.

93. The pt. Q lies in the plane rRr′, Fig. 5. Its V proj., *q*′, is given. Find *q*, and give its dist. from the G. L.

94. The V proj. of a pt. Q lying in the plane tTt′ is given at *q*′, Fig. 5. Find *q*, and give its dist. from the ground line. Draw double size.

95. The line QO lies in the plane rRr′, Fig. 5. Its V proj., *q*′*o*′, is given. Locate *qo*.

96. If the line QO lies in the plane sSs′ and *q*′*o*′ is given, Fig. 5, locate *qo*.

*97. If *ab* is the H proj. of a line lying in the plane rRr′, Fig. 4, locate *a*′*b*′.

*98. If *Gm* is the H proj. of a line lying in rRr′, Fig. 4, locate *g*′*m*′. (The pt. G lies in the ground line.)

*99. If *bcd* is the H proj. of a triangle BCD lying in the plane sSs′, Fig. 6, find its V proj. and the true size and shape of the triangle.

100. Thru D, pass a line, MS, ⊥ to uUu′, Fig. 5. Const. double size.

101. Thru CD, pass a plane, tTt′, ⊥ to rRr′, Fig. 4. Give the dist. RT.

102. Thru BM, Fig. 6, pass a plane, uUu′, ⊥ to sSs′. Give dist. US.

103. Thru EM, Fig. 4, pass a plane sSs′, ⊥ to kKk′. Give dist. SK.

104. Find the dist. from the pt. D to the plane uUu′, Fig. 5.

105. Find the dist. from the pt. E to the plane sSs′, Fig. 6.

---

*In solving Probs. 97, 98 and 99, no attention should be paid to the V proj's. as given in the figure.

106. Find the dist. from the pt. D to the plane uUu′, Fig. 7.

107. Find the dist. from the pt. D to the plane tTt′, Fig. 5. Draw double size.

108. Find the dist. from the pt. N to the plane sSs′, Fig. 5.

109. Find the dist. from the pt. D to the plane tTt′, Fig. 7.

110. Find the dist. from the pt. N to a plane thru W, ⊥ to the G. L., Fig. 5.

111. Find the dist. from one corner of a 3″ cube to the plane of the three adjacent corners.

112. Find the dist. from one corner of a 2½″ cube to the plane of the three adjacent corners.

113. Project the line DN upon the plane uUu′, Fig. 5. and show the true position of the proj. D_1N_1, with respect to the H trace.

114. Project the line BE, Fig. 6, upon the plane rRr′, and show the true relation of this proj. B_1E_1, to the H trace.

115. Project the line DN, Fig. 5, upon the plane sSs′, and show the true relation of this proj. D_1N_1, to the H trace.

116. Project the line DN, Fig. 5, upon a plane thru W, ⊥ to the G. L., and show the true relation of this proj. D_1N_1, to the V trace. Draw double size.

117. Project the triangle BMN, Fig. 6, upon the plane sSs′, and show the true form of this proj. $B_1M_1N_1$, giving ∠ $M_1N_1B_1$.

118. Project the triangle ABG, Fig. 8, upon the plane sSs′, and show the true form of this proj. $A_1B_1G_1$, giving ∠ $G_1A_1B_1$.

119. Thru K, Fig. 4, const. the traces of a plane ‖ to rRr′. Find the dist. bet. these two planes. Const. double size.

120. Thru K, Fig. 4, const. the traces of a plane ‖ to gGg′. Find the dist. bet. the two planes.

121. Thru C, Fig. 4, pass a plane tTt′ ⊥ to the line MD. Give dist. KT.

122. Thru M, Fig. 4, pass a plane, tTt′, ⊥ to the line BS. Give dist. KT.

123. Thru D, Fig. 8, pass a plane, uUu′, ⊥ to the line GO. Give dist. SU.

124. Thru C, Fig. 4, pass a plane tTt′, ⊥ to the line AS. Give dist. bet. H and V traces on drawing.

125. Thru G, Fig. 8, pass a plane uUu′, ⊥ to the line AC. Give dist. bet. H and V traces on drawing.

126. Thru D, Fig. 4, pass a plane tTt′, ⊥ to the line BC. Give dist. RT.

127. Const. the proj.s of a circle whose center lies in the line MD, Fig. 4, whose plane is ⊥ to MD, and whose circumference passes thru B. Give major and minor axes of H proj.

128. Thru D, Fig. 4, pass a plane, tTt′, || to the lines BC and EM. Give dist. GT.

129. Thru N, Fig. 6, pass a plane uUu′, || to the lines BM and CE. Give dist. TU.

130. Thru O, Fig. 8, pass a plane uUu′, || to the lines GM and DE. Give dist. SU.

131. Thru D, Fig. 4, pass a plane tTt′, || to the lines AB and SM. Give dist. bet. H and V traces on drawing.

132. Thru D, Fig. 6, pass a plane tTt′, || to the lines AB and CE. Give dist. bet. H and V traces on drawing.

133. Thru B, Fig. 4, pass a plane sSs′, || to the lines CD and SM. Give dist. KS.

134. Thru B, Fig. 6, pass a plane uUu′, || to the lines MD and AE. Give dist. SU.

135. Thru D, Fig. 4, pass a plane, tTt′, || to rRr′. Give dist. RT.

136. Thru N, Fig. 5, pass a plane, kKk′, || to uUu′. Give dist. UK.

137. Thru M, Fig. 4, pass a plane, sSs′, || to kKk′. Give dist. SK.

138. Pass a plane, sSs′, || to rRr′, Fig. 4, and 1¼″ from it. Give dist. SR.

139. Pass a plane, kKk′, || to uUu′, Fig. 5, and 1½″ from it. Give dist. KU.

140. Pass a plane, tTt′, || to kKk′, Fig. 4, and 1″ from it. Give dist. TK.

141. Pass a plane, tTt′, thru the line SB, || to MD, Fig. 4. Give dist. KT.

142. Pass a plane, kKk′, thru the line CB, || to MN, Fig. 6. Give dist. KS.

143. Pass a plane, tTt′, thru the line MD, || to AB, Fig. 4. Give dist. bet. traces on drawing.

144. Pass a plane, sSs′, thru the line AB, || to SM, Fig. 4. Give dist. bet. traces on drawing. Draw double size.

145. Pass a plane, tTt′, thru the line AS, || to MC, Fig. 4. Give dist. KT.

146. Pass a plane, uUu′, thru the line MC, || to AS, Fig. 4. Give dist. KU.

147. Find the shortest dist. from the pt. C to the line MD, Fig. 4.

148. Find the shortest dist. from the pt. M to the line DN, Fig. 6.

149. Find the shortest dist. from the pt. D to the line AB, Fig. 8.

150. Find the shortest dist. from the pt. C to the line BD, Fig. 4.

151. Find the shortest dist. from the pt. M to the line AS, Fig. 4.

152. Find the shortest dist. from the pt. S to the line AB, Fig. 4.

153. Find the shortest dist. from the pt. B to the line SE, Fig. 4.

154. Find the dist. bet. the || lines DN and EO, Fig. 6.

155. Find the ∠ which the line DN, Fig. 5, makes with the plane uUu′.

156. Find the $\angle$ which the line DG, Fig. 8, makes with the plane sSs′.

157. Find the $\angle$ which the line DN, Fig. 5, makes with the plane sSs′. Draw double size.

158. Find the $\angle$ which the line DN, Fig. 5, makes with a plane thru W, $\perp$ to the G. L.

159. Find the $\angle$ which the line AB, Fig. 4, makes with the plane rRr′.

160. Find the $\angle$ which the line AS, Fig. 4, makes with the plane rRr′.

161. Find the intersection of the planes gGg′ and rRr′, Fig. 4.

162. Find the intersection of the planes rRr′ and sSs′, Fig. 6.

163. Find the intersection of the planes gGg′ and kKk′, Fig. 4.

164. Find the intersection of the planes rRr′ and tTt′, Fig. 6.

165. Find the intersection of the planes sSs′ and tTt′, Fig. 6. Draw double size.

166. Find the intersection of the planes tTt′ and sSs′. Fig. 5. Give dist. bet, proj.s on drawing.

167. Find the intersection of the planes sSs′ and tTt′, Fig. 7. Give dist. bet. proj.s on drawing.

168. Find the intersection of the planes uUu′ and rRr′, Fig. 5.

169. Find the intersection of the planes uUu′ and rRr′, Fig. 7.

170. Find the intersection of the plane uUu′, Fig. 5, with a plane || to H whose V trace is Tt′.

171. Find the intersection of uUu′, Fig. 5, with a plane containing the G. L., making an $\angle$ of 30° with H, and passing thru the 1st and 3rd quadrants.

172. Pass a line thru M, Fig. 4, and || to both gGg′ and rRr′.

173. Pass a line thru D, Fig. 8, and || to both sSs′ and tTt′.

174. Find the ∠ bet. the planes gGg′ and rRr′, Fig. 4.

175. Find the ∠ bet. the planes rRr′ and sSs′, Fig. 6.

176. Find the ∠ bet. the planes sSs′ and tTt′, Fig. 8.

177. Find the ∠ bet. the planes sSs′ and tTt′, Fig. 5.

178. Find the ∠ bet. the planes sSs′ and tTt′, Fig. 7.

179. Find the ∠ bet. the planes sSs′ and rRr′, Fig. 5.

180. Find the ∠ bet. the planes tTt′ and uUu′, Fig. 5.

181. Find the ∠ bet. the plane rRr′, Fig. 4, and each plane of proj.

182. Find the ∠ bet. the plane uUu′, Fig. 5, and each plane of proj.

183. Find the ∠ bet. the plane kKk′, Fig. 4, and each plane of proj.

184. If *bc,* Fig. 4, is the V trace of a plane, tTt′, that makes an ∠ of 75° with V, const. the H trace of the plane. (Two solutions.) Give the ∠ bet. H trace and G. L.

185. If *wn,* Fig. 6, is the H trace of a plane, uUu′, that makes an ∠ of 60° with H, what ∠ does the plane make with V?

186. If *sc,* Fig. 4, is the H trace of a plane, tTt′, that makes an ∠ of 45° with H, const. the V trace, giving its ∠ with G. L. (Two solutions.)

187. If *d′m′*, Fig. 8, is the V trace of a plane, kKk′, that makes an ∠ of 45° with V, what ∠ does the plane make with H?

188. If the V trace of a plane, sSs′, is ⊥ to the G. L., and the plane makes an ∠ of 60° with V, const. the H trace. (Two solutions.)

189. If *ab,* Fig. 4, is the H trace of a plane, uUu′, that makes an ∠ of 60° with H, and traverses the 3rd quadrant, const. the V trace, giving its dist. from the G. L.

190. If *we,* Fig. 6, is the H trace of a plane kKk′ that traverses the 4th quadrant and makes an ∠ of 52½° with H, const. the V trace, giving its dist. from the G. L.

191. Find the pt. F, in H, equally dist. from D, W and O, Fig. 8.

192. Find a pt. W in V, equally dist. from S, M and D, Fig. 4.

*193. Take *dn* as the H proj. of a line lying in the plane uUu′, Fig. 5. Find *d′n′*, and const. a line DC lying in uUu′, making 60° with DN. Give ∠ *c′d′n′*. (Two solutions.)

*194. Take *a′m′* as the V proj. of a line lying in sSs′, Fig. 6. Find *am*, and const. a line AC lying in sSs′ and making 45° with AM. Give ∠ *cam*. (Two solutions.)

195. Find the shortest line OQ that can connect the lines MD and BR, Fig. 4. Give its length. Const. double size. (The pt. R is in the G. L.)

196. Find the shortest line, FJ, connecting the lines GO and MD, Fig. 8. Give its length.

197. Find the shortest line, FJ, that can be drawn bet. the lines ME and DN, Fig. 8. Give its length.

198. Find the shortest line, OQ, connecting the lines AB and EC, Fig. 4. Give its length.

199. Find the shortest line, OQ, conecting the lines AE and MD, Fig. 4. Give its length.

200. Find the shortest line, OQ, connecting CD and EM, Fig. 4. Give its length.

MISCELLANEOUS PROBLEMS ON THE POINT, LINE AND PLANE, FOR ORIGINAL SOLUTIONS

201. The V proj. of a line ⊥ to DN, Fig. 5, is *q′d′*. Find the H proj.

202. *cw* is the H proj. of a line ⊥ to CE, Fig. 6. Find *c′w′*.

---

*In solving Prob's 193 and 194, no attention should be given to *d′n′* or *am* as given in the Fig's.

203. In the above problem, taking CE and CW as two sides of a rectangle, complete the rectangle ECWF and obtain by points the projs. of the inscribed ellipse.

204. Obtain by points the projs. of the parabola inscribed in the above rectangle.

205. Show the projs. of the circle inscribed in the triangle ABD, Fig. 8.

206. Show the projs. of the circle circumscribed about the triangle ABD, Fig. 8.

207. Thru B, Fig. 4, const. a line, BF, making 20° with H and 40° with V.

208. Thru N, Fig. 5, const. a line NK making 22½° with H and 30° with V.

209. A pt. M is in the 1st quadrant, 1.5″ from H, and 2″ from V. A pt. N is in the 1st quadrant 0.5″ from H and 3″ from V. The length of the line MN is 4″. Determine the projs. of the line MN.

210. A rod 2½ feet long is suspended horizontally by vertical threads 3 feet long attached to its ends. How far will the rod be raised by turning it thru 90°?

211. How far will the above rod be raised by turning it thru 60°?

212. How far will the above rod be raised by turning it thru 120°?

213. The H trace of a plane tTt′ makes an ∠ of 30° with the G. L. The plane makes an ∠ of 45° with V. What ∠ does the V trace make with the G. L.?

214. The V trace of a plane, sSs′, makes 45° with the G. L. The plane makes 60° with H. What is the tangent of the ∠ bet. the H trace and the G. L.?

215. The H trace of a plane, tTt′, makes 60° with the G. L. The ∠ bet. the two traces *in space* is 75°. What ∠ does the V trace make with the G. L.?

216. The H trace of a plane sSs′ makes an ∠ of 67½° with the G. L. The ∠ bet. the two traces *in space* is 105° What is the tang. of the ∠ bet. the V trace and the G. L.?

217. A pt. Q lies in the plane sSs′, Fig. 6. Its position when developed into H is at *d*. Determine the two projs. of the pt.

218. The pt. N, Fig. 7, lies in a plane kKk′. Its developed position in H is at *d*. Const. the traces of the plane kKk′. The H trace passes *between n* and *d*.

*219. Take *b* as the H proj. of a pt. lying in rRr′, Fig. 8. Thru this pt. const. a line, BK, making an ∠ of 30° with H, and lying in the plane.

220. Const. a line, CE, in the H plane, making an ∠ of 60° with DN, Fig. 7. Give ∠ bet. CE and *dn*.

221. Given *bd*, Fig. 8, as the H trace of a plane kKk′, and the pt. G, ¾″ dist. from the plane, const. the V trace. (Hint: Thru the pt. G pass a plane ⊥ to the given trace and revolve it into the corresponding plane of proj.) Two solutions.

222. Thru B, Fig. 8, const. a line BF touching the lines GM and DO.

223. Thru D, Fig. 4, const. a line DN touching AE and BC.

224. Thru E, Fig. 6, pass a plane, kKk′, ∥ to BM and ⊥ to sSs′. Give dist. KS.

225. The line OQ, Fig. 7, lies in uUu′. Const. its H proj., and thru OQ pass a plane kKk′ making an ∠ of 45° with uUu′. (Two solutions.)

226. Thru CD, Fig. 8, pass a plane kKk′, making an ∠ of 45° with rRr′. (Two solutions.)

227. Thru D, Fig. 5, pass a line DA making an ∠ of 60° with rRr′ and cutting the line OQ, which lies in rRr′.

228. A ray of light, ND, Fig. 5, proceeding from the point N strikes the plane uUu′. Show the pts. 1, 2, 3 and 4 where it is successively reflected from uUu′ and the three planes of proj. Let the P plane pass thru N. Const. double size.

---

*In Prob. 219, no attention should be paid to the V proj. of B, as given in the figure.

229. Of a plane pentagon BADGN, suppose we have given *badgn* and *b'a'd'*, Fig. 8. Find the pts. *g'* and *n'*, and the true figure, *without* constructing the plane of the pentagon. (No attention should be paid to the V projs. of G and N as given in the figure.)

*230. Two sides of a plane rectangle are horizontally projected in *ab* and *ad*, Fig. 8. A lies in H, and the true length of AB is 2″. Const. the V proj. of the rectangle.

231. Pass a line FK, || to AB, Fig. 8, and cutting DG and WO.

232. Const. the projs. of a regular hexagon whose center is D, Fig. 8, and one of whose sides lies in AB.

233. Const. a line, FJ, lying in the plane sSs′, Fig. 8, and ⊥ to DG.

234. Assume a 2″ cube in 1st quadrant with faces || to, and dist. 1¼″ from H and V. Required the projs. of the solid formed by passing planes thru the 12 edges of the cube, each ⊥ to the diagonal plane in which the edge lies.

235. Bisect the ∠ bet. the planes rRr′ and sSs′, Fig. 6, by a plane, kKk′. Draw double size.

236. Find a point, N, in the line SB, Fig. 4, equally dist. from the planes gGg′ and rRr′.

237. Thru A, Fig. 4, pass a plane sSs′, making an ∠ of 75° with H and 45° with V. (Four solutions.)

238. Thru E, Fig. 6, pass a plane tTt′, making an ∠ of 60° with H and 67½° with V. (Four solutions.)

239. Const. the three projs. of a regular hexagonal prism 3½″ high, each side of whose base is 1″, the base making an ∠ of 30° with H and 75° with V, and one edge of the base being || to H; the center, C, being ¾″ from H and 1½″ from V.

240. Const. the projs. of a 1½″ cube with one face in H and one face making an ∠ of 30° with V.

---

*In Prob. 230, no attention should be paid to *a′*, *b′* or *d′* as given in the figure.

241. Const. the projs. of a 1½″ cube, one edge in H making 30° with the G. L. and one face inclined 30° to H on side *away* from the G. L.

242. Const. the projs. of a 1½″ cube, one "body diagonal" being ⊥ to H and the H proj. of one edge being ⊥ to the G. L.

243. A cube lies with one of its faces in uUu′, Fig. 5. The V proj. of the lowest edge is *d′n′*. Const. the three projs. of the cube, taking the P plane thru W. Const. to scale 1½″=1″.

244. Const. the projs. of a cone of revolution whose base lies in rRr′, Fig. 8, whose vertex is at A, and whose elements make an ∠ of 22½° with the axis. Const. double size.

245. Const. the projs. of a cone of revolution whose vertex is at A, Fig. 8, whose base has a dia. of 1½″ and lies in the plane rRr′. Const. double size.

246. Rotate the pt. D, Fig. 8, about the line AW, and find the points where it passes thru H, V and P. (Take P thru the pt. B.) Also show the position of D after it has rotated thru 90° from its original position.

247. Rotate AB, Fig. 8, thru 90° about CD. Show its new projs. (Two solutions.)

248. Const. a line, FJ, || to rRr′, Fig. 8, at a dist. 1″ from it, and cutting the lines AB and CD. (Two solutions.)

249. Thru D, Fig. 4, pass a line, DF, ⊥ to MD and || to rRr′.

250. Pass a plane, sSs′, twice as far from S as from D, M and C, Fig. 4. (Two solutions.)

251. Pass seven planes, each equally dist. from S, D, M and C, Fig. 4.

252. Pass a plane, tTt′, twice as far from S and M as from D and C, Fig. 4. (Two solutions.)

253. Pass a plane, sSs′, || to both AB and CD, Fig. 8, and equally dist. from each.

254. Pass a plane, tTt′, || to both AB and CD, Fig. 8,

and twice as far from AB as from CD. (Two solutions.)

255. The point G, Fig. 8, lies in a plane kKk′ whose H trace is *bd*. (a) Find the V trace, giving the ∠ bet. Kk′ and the G. L. (b) Const. the projs. of a sphere of 1″ radius, tang. to kKk′ at the pt. G. (Two solutions.)

256. Given WNE, Fig. 8, as the base of a triangular pyramid lying in H, and the lateral edges, WF=1¼″, NF=1⅛″ and EF=1″, const. the projs. of the pyramid double size. Give height of F above H.

257. Given WEB, Fig. 8, as the base of a triangular pyramid lying in H, and the lateral edges, WF=2″; EF=1¾″; BF=2¼″, const. the projs. of the pyramid double size. Give height of F above H.

258. Given BWN, Fig. 8, as the base of a triangular pyramid lying in H, and the lateral edges, BF=2½″, WF=1½″, NF=2″, const. the projs. of the pyramid. Give height of F above H.

259. The base of a triangular pyramid has its sides, BC=3″, BD=2.5″, CD=2.25″. The lateral edges are, AB=2¾″, AC=2⅝″, AD=2¼″. Const. the pyramid, giving its altitude.

260. Circumscribe a sphere about the pyramid in the preceding problem. Give radius.

261. Circumscribe a sphere about the triangular pyramid D—BWN, Fig. 8. Give (a) dist. of center above H, (b) radius of circumscribed sphere.

262. Circumscribe a sphere about the triangular pyramid O—WEN, Fig. 8, Give (a) dist, of center above H, (b) radius of circumscribed sphere. Const. double size.

263. Pass a sphere thru the four pts. B, D, G and W, Fig. 8. Give radius of sphere and dist. of center above H.

264. Inscribe a sphere in the triangular pyramid D—BWN, Fig. 8. Give radius of sphere. Const. double size.

265. Inscribe a sphere in the triangular pyramid of Prob. 259. Const. double size, and give radius of sphere.

TRIHEDRAL ANGLES—GRAPHICAL SOLUTION OF SPHERICAL TRIANGLES

In the following problems, let A, B and C represent the dihedral angles (or spherical angles), and a, b and c the face angles (or sides of the spherical triangle) opposite A, B and C respectively.

266. Given $b=63°15'$; $c=47°42'$; $C=55°53'$; find A, B and a.

267. Given $a=79°1'$; $b=82°17'$; $A=82°9'$; find B, C and c.

268. Given $a=64°47'$; $c=48°3'$; $C=54°8'$; find A, B and b.

269. Given $a=38°$; $b=42°$; $B=58°53'$; find A, C and c.

270. Given $B=82°$; $C=67°$; $b=73°48'$; find a, c and A.

271. Given $A=59°4'$; $B=88°12'$; $a=50°2'$; find b, c and C.

272. Given $A=51°58'$; $C=83°55'$; $c=51°$; find a, b and B.

273. Given $A=73°47'$; $B=54°8'$; $a=61°47'$; find b, c and C.

274. Given $a=61°47'$; $b=43°3'$; $C=80°20'$; find A, B and c.

275. Given $a=79°1'$; $b=82°17'$; $C=45°44'$; find A, B and c.

276. Given $b=63°15'$; $c=47°42'$; $A=59°4'$; find B, C and a.

277. Given $a=64°47'$; $b=61°47'$; $C=54°8'$; find A, B and c.

278. Given $B=90°$; $C=45°44'$; $a=79°1'$; find A, b and c.

279. Given $B=88°12'$; $C=55°53'$; $a=50°2'$; find b, c and A.

280. Given $A=83°55'$; $B=51°58'$; $c=42°$; find a, b and C.

281. Given $B=73°47'$; $C=54°8'$; $a=64°47'$; find b, c and A.

282. Given $a=51°$; $b=38°$; $c=42°$; find A, B and C.

283. Given $a=63°12'$; $b=68°46'$; $c=73°48'$; find A, B and C.

284. Given $a=47°42'$; $b=50°2'$; $c=63°15'$; find A, B and C.

285. Given $a=48°3'$; $b=64°47'$; $c=61°47'$; find A, B and C.

286. Given $A=83°55'$; $B=51°58'$; $C=58°53'$; find a, b and c.

287. Given $A=54°8'$; $B=80°20'$; $C=73°47'$; find a, b and c.

288. Given $A=45°44'$; $B=82°9'$; $C=90°$; find a, b and c.

289. Given $A=59°4'$; $B=88°12'$; $C=55°53'$; find a, b and c.

DETERMINATION OF SKEW ANGLES IN ROOF STRUCTURES

An application of Descriptive Geometry to the solution of practical problems involving the right line and plane is shown in the determination of skew angles in beam connections for roofs or other structures. The presentation of the subject here given is a modification of that given by Prof. C. G. Wrentmore, and published in *The Michigan Technic* of 1903. For the solution of the same problem by spherical trigonometry, see Mr. C. A. P. Turner's article in Eng. News of Feb. 15 and 22, 1900.

Since the graphical method insures against the placing of two bodies in the same place at the same time, it is considered preferable to the method of computation. The experience of those who have tried both methods has shown that the graphical method presents less probability of error and greater probability of all clearances being provided for,

than the method by computation; and where the principal dimensions are checked by computation, a drawing to quarter size will give all secondary dimensions with sufficient accuracy for all cases, both for angular and linear measurements.

Drawings for this purpose, however, must be done with exactness; and the draftsman must know his own limitations in the matter of accurate work before he can determine how far his graphical solutions are trustworthy.

Fig. 9 is a perspective view of several roofs forming *hips* and *valleys*. In roofs R, R′ and R″, horizontal lines run at right angles; but in roofs R″, R‴, etc., horizontal lines run at an angle of 135°.

Fig. 10 is a descriptive drawing for the solution of angles connected with roofs R and R′. The planes of pro-

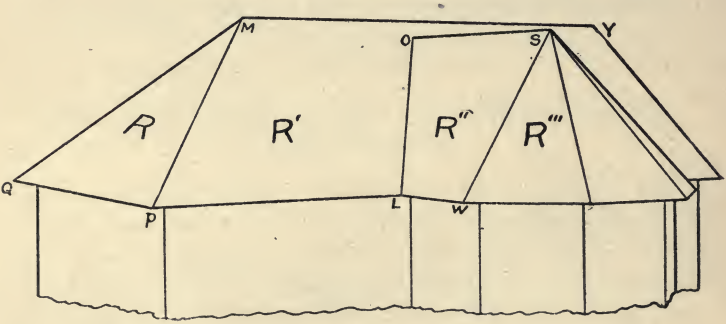


Fig. 9.

jection used are, the horizontal plane, two vertical planes respectively ⊥ to the H traces of the roof planes, and another vertical plane through the hip MP. We have then three ground lines: XX′, *op″* and *mp*. The V plane that is used for any particular solution is theoretically immaterial; but practically much time and labor may be saved by choosing the best one.

In these problems the *pitch* of each roof (height divided

by span) will be given, and the following angles will be worked out, the same general method being used whether the roofs form a hip or a valley, or whether the eaves run at right angles or otherwise.

A is the angle bet. plane of roof and the H plane

We choose the pt. p (Fig. 10) in any convenient part of the sheet, and draw the H traces pp' and pp'' of the two roofs. It is evident that the tangent of the $\angle$ A is double the pitch. If then the pitch of the roof R is given as ¼, the $\angle$ A will have a tangent of ½. We lay off two units to the right of p' and one unit up, giving us the point m'. The V trace of the roof plane R is $p'm'$, and the required $\angle$ is A. Similarly we obtain A′ from the given pitch of R′ (in this case 1/3).

B is the angle bet. the H trace of the roof plane, and the H trace of the hip-web plane, or valley-web plane

Assume any pt. m'' in the roof line $p''m''$ at a height nm'' above H, and find the pt. m' in the other roof line at the same height, by taking $ym'=nm''$. Draw lines thru y and n || to the eave lines pp' and pp'' respectively. They meet in m; and pm is the H proj. of the hip. It is also the H trace of the hip-web plane, and $p'pm$ is the $\angle$ B for roof R; and $p''pm$ is the $\angle$ B′ for the roof R′.

C is the angle bet. the hip (or valley) rafter, and the H plane

Revolve the H projecting plane of the hip into H. The pt. M will fall at m''', whose dist. from m is equal to ym'. The revolved position of the hip is pm''', and mpm''' is the $\angle$ C.

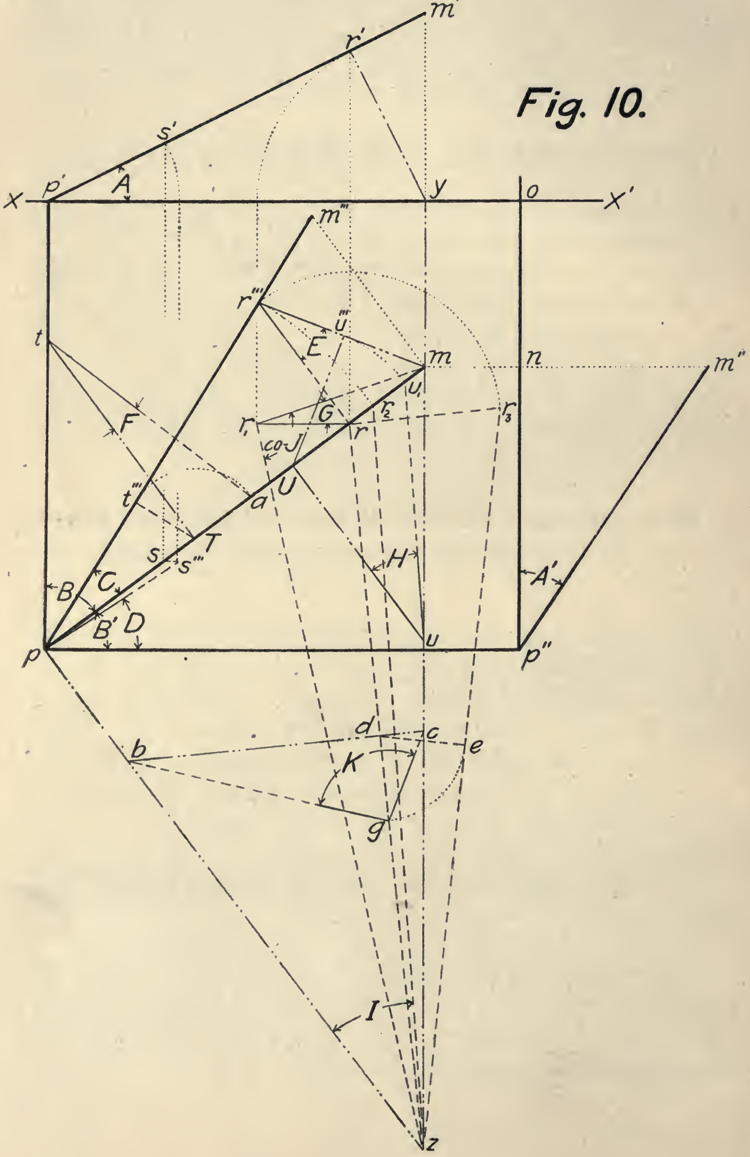

m'
r'
s'
p'
A
x
y
o
x'
m'''
r'''
u'''
E
t
m
n
m''
u,
G
r₂
r₃
r,
r
F
co-J
U
a
t'''
T
H
s
s'''
A'
B
C
B'
D
p
u
p''
d
c
e
b
K
g
I
z

Fig. 10.

D is the angle in the roof plane bet. a main rafter and the hip rafter (or valley rafter)

Revolve the roof plane about its H trace pp' into H. Any pt. in the hip line, as S, will fall into the position s''', and the hip will fall into the position ps'''. Any main rafter is $\perp$ to pp' and falls $\|$ to pp''. Hence $s'''pp''$ is the value of D. Proceed in a similar manner for the $\angle$ D′.

E is the angle bet. a vertical line and the trace of the purlin web upon the hip web plane

The plane of the purlin web is taken normal to the plane of the roof, its H trace being my, its V trace yr' and its trace on the hip web plane mr'''. This trace makes with the vertical line rr''' the $\angle$ E required. Similarly for the $\angle$ E′.

F is the angle bet. the roof plane and plane of back of hip (or valley)

These two planes intersect in the hip line PM. The H trace of the back of hip is pz. Pass a plane tTt‴ normal to the hip. Its H trace is Tt, normal to pm. Its trace on the hip web plane is Tt‴, normal to pm'''. It cuts from the back of hip a line normal to pm, and from the plane of R a line which, when revolved about Tt into H, falls into the position at. The $\angle$ F is therefore the $\angle$ bet. at and a line normal to pm, or the $\angle$ Tta.

G is the angle in purlin-web bet. a normal to center line of purlin, and the trace of hip-web on purlin-web

The intersection of hip and purlin webs is horizontally projected in mr. Revolving the purlin web plane about its H trace, my, the point R falls at r_1, and the line mr takes the position mr_1. The required $\angle$ is mr_1r.

H is complement of angle bet. purlin-web and hip-web planes

A plane ⊥ to the intersection (MR) of purlin-web and hip-web is assumed at uUu‴. This cuts from the purlin-web a line which revolves into H at uu_1, and from the hip-web a line which revolves into H in Uu_1. The ∠ bet. the planes is therefore shown at uu_1U, and its complement at u_1uU.

I is the angle in back of hip bet. a line normal to hip-web and the trace of purlin-web on back of hip

The H trace of back of hip is *pz*. This plane is cut by the purlin web in the line horizontally projected at *rz*. When this line is revolved about *pz*, it falls at r_2z. A line normal to the hip web will revolve into H in a parallel to *pz*. The ∠ pzr_2 is therefore the ∠ I required.

J is complement of angle in purlin-web bet. traces of hip-web and back of hip

The line *rz* revolved about *yz* into H falls at r_1z. The revolved position of *mr*, as already found, is mr_1. The ∠ bet. these lines, mr_1z, is therefore the complement of the ∠ J, which in this case is a negative angle.

K is angle bet. back of hip and purlin web

The line of intersection of the two planes is RZ. The H projecting plane of this line is revolved about its H trace, *rz*, into H; the line RZ then falling at r_3z. A plane normal to both planes will have its H trace at *bc*, ⊥ to *rz*; and its trace upon the H projecting plane of RZ is shown in revolved position at *de*. If the plane *bc* be now revolved about *bc* into H, the pt. where it cuts the line RZ falls at *g*, and the ∠ bet. the planes is shown at *bgc*.

290. Determine graphically the eleven $\angle^s$, A′ to K′ inclusive, for the roof R′, when the pitch of R is ¼, and that of R′ is 1/3.

291. Determine the $\angle^s$ A′ to K′ for the roof R′ when pitch R′=1/3, pitch R″=½, and the two roofs form a valley with eaves at rt. $\angle^s$.

292. Determine the $\angle^s$ A″ to K″ for the roof R″ in the above case.

293. Determine the $\angle^s$ A″ to K″ for the roofs R″ and R‴ when each has a pitch of ½, and the eaves form an $\angle$ of 135°.

294. Determine the $\angle^s$ A″ to K″ for the roof R″ when pitch R″=1/3 and pitch R‴=2/5. Roofs form a hip, and eaves run at an $\angle$ of 120°.

295. Determine the $\angle^s$ A‴ to K‴ for the roof R‴ in the above case.

296. Determine the $\angle^s$ A to K for the roof R, when pitch R=2/5, pitch R′=½, roofs form a hip and eaves form a rt. $\angle$.

297. Determine the $\angle^s$ A′ to K′ for the roof R′ in the above case.

298. Determine the $\angle^s$ A′ to K′ for the roof R′, when pitch R′=¼, pitch R″=2/5, roofs form a valley and eaves run at rt. $\angle^s$.

299. Determine the $\angle^s$ A″ to K″ for the roof R″ in the above case.

REPRESENTATION OF SURFACES AND SPACE CURVES

300. Const. the H and V projs. of one complete convolution of a helix with axis $\perp$ to H. Generating point moves 4″ in direction of axis for each revolution about the axis. Dist. of generating pt. from axis 1½″.

301. A right circular cone of 3½″ alt., and dia. of base 3″, is resting with its base in H. Another rt. circular cone, with equal elements, and dia. of base 1½″, has its vertex

coincident with that of the first cone, and an element in common with it. Const. the H and V projs. of the *spherical epicycloid* generated by a pt. in the circumference of the base of the second cone as it rolls upon the first.

302. Const. the H and V projs. of a cone of revolution with base lying in H, and vertex in 1st quadrant. Dia. of base, 2″; alt. of cone, 3″. Assume an element, SA, of the cone, and show the inclination of the elements to H.

303. Const. the H, V and P projs. of an oblique cone with an elliptical base lying in H, and with axis in 1st quadrant oblique to H, V and P. Assume a point, A, on the surface.

304. A right circular cone with axis ON, Fig. 8, base in H, and dia. of base 1¼″, is in contact with an equal cone lying on an element in H, and with its vertex at E. Const. the projs. of the two cones.

305. Const. the projs. of a cyl. of revolution, base in H, axis in V, dia. of base 1″, elements inclined 45° to H.

306. Const. the projs. of an oblique cyl. with circular base lying in H; elements || to V, and inclined 60° to H; dia. of base, 1¼″. Assume an element AB and a pt. C, on the surface.

307. Assuming the ellipse in Fig. 12, as the base of a cyl. of revolution with elements || to V, const. the V proj. of the cyl. and give the inclination of the elements to H.

308. Assume a point, A, on the surface of a helicoid whose elements are inclined at an angle of 60° to the axis. Generating pt. of helix moves 4″ in direction of axis for each revolution about the axis. Dist. of generating pt. from axis, 1¾″.

309. Given CD and GM, Fig. 8, as the directrices of an hyperbolic paraboloid, with rRr′ as plane directer, assume an element of the surface through G. Assume also a point F on that element.

310. Given GO and DW, Fig. 8, as the directrices of

an hyperbolic paraboloid, the plane directer being the H plane, const. an element thru D, and, one thru W, and assume a point F on the first element.

311. Given CD and GM, Fig. 8, as the directrices of an hyperbolic paraboloid, and DG and CM as two elements of the surface, find a plane directer for each generation.

312. Assume a hyperbolic paraboloid with CD and WO, Fig. 8, as directrices, and CO and DW as two elements. The H proj. of a pt. F on the surface is 1⅛″ below S. Find *f′* and give its dist. from the G. L.

313. With *d*, Fig. 8, as center, strike an arc to the left with radius 2¼″. With *d′* as center and radius 2¼″, strike an arc to the left. These are the H and V projs. of a curved line. With *c′* as center and radius 3″, strike an arc to the right and below the G. L., and with *a′* as center, and 3″ radius, strike an arc to the right and above the G. L. These are the H and V projs. of another curved line. Taking these as the directrices of a warped surface with rRr′ as the plane directer, const. the projs. of an element of the warped surface thru a pt. in the left hand curve 1″ above H.

314. Const. the two projs. of an hyperboloid of revolution of one nappe, with ON, Fig. 8, as axis, O the center of the gorge circle whose dia. is 1″, and elements inclined 60° to H. Assume a point, Q, on the surface.

315. Const. the projs. of the right and left hand elements of a conoid with AD, Fig. 8, as one directrix, and a circle in H with center at B and 1½″ dia., as the other directrix; the plane directer being sSs′. Assume a point Q on the surface.

316. The three directrices of an hyperboloid of one nappe are BD, WO, and GN, Fig. 8. Show the projs. of an element thru B, and of one thru O.

317. Assume a point, K, on a sphere with center at O, Fig. 8, and dia. 1½″.

318. A prolate ellipsoid of revolution has its axis in

ON, Fig. 8, and its center at O. The axes of the revolving ellipse are 2½″ and 1½″. Show the projs. of the ellipsoid, and assume a point, F, on the surface.

319. Taking the helix in Prob. 300 as the directrix of a helical convolute, assume an element of the surface and a point K on the surface. Also const. the trace of the surface on the H plane.

320. Assume a helix with axis ⊥ to the G. L.; pitch 3½″; dia. of H proj. 3½″. Taking this as the directrix of a helical convolute, assume an element of the surface, and a point on that element, and const. a portion of the trace of the surface on the H plane.

PROBLEMS RELATING TO LINES AND PLANES TANGENT TO SINGLE CURVED SURFACES

321. Thru A, Fig. 11, on the surface of the left hand cyl., pass a plane, tTt′, tang. to the cyl. The pt. A is on the side nearest the V plane. Give ∠ RTt′.

322. The pt. B, Fig. 11, is on the lower side of the cyl. Pass a plane sSs′, thru B, tang. to the cyl. Give ∠ RSs′.

323. The pt. E. is on the upper side of the right hand cyl. in Fig. 11. Pass a plane, tTt′, thru E, tang. to the cyl. Give ∠ RTt′.

324. Take the ellipse in Fig. 12 as the base of a cyl. with elements || to AB. Pass a plane, tTt′, thru E and tang. to the cyl.; the point being on that portion of the surface of the cyl. nearest V. Give ∠ RTt.

325. Take the left hand circle in Fig. 11 as the base of a cyl. with elements || to V and inclined 60° to H. Pass a plane sSs′ thru A and tang. to the cyl.; the pt. A being on that portion of the cylindrical surface nearest V. Give ∠ RSs′. Draw double size.

326. Take BN, Fig. 12, as the axis of a cyl. of revolution with dia. 1″. Thru C, on the upper side, pass a

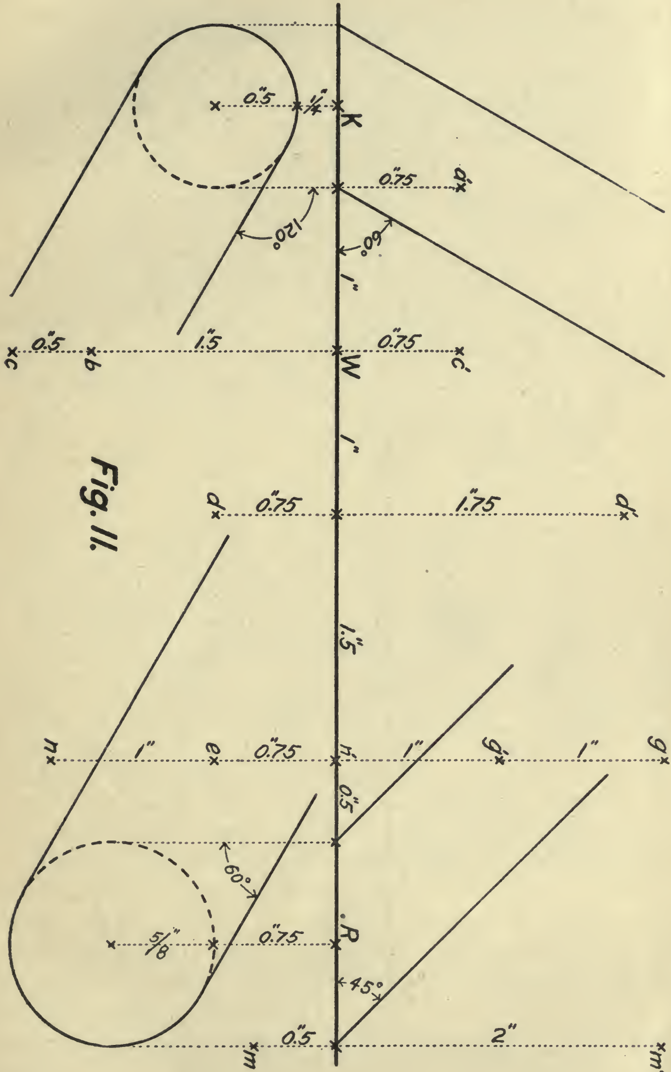


Fig. 11.

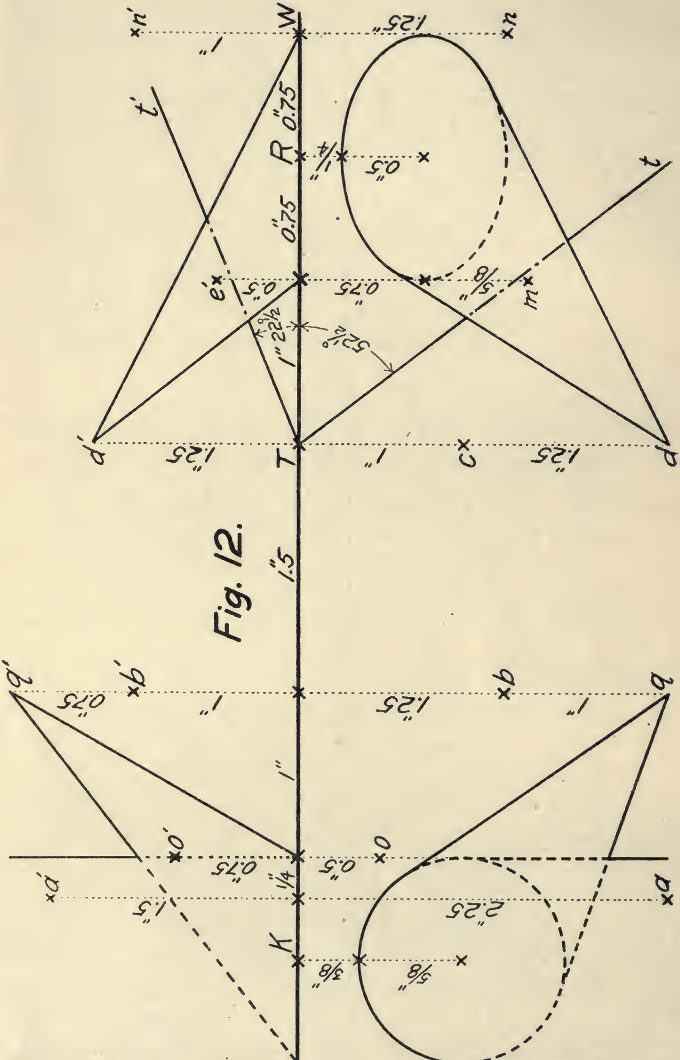


Fig. 12.

plane, tTt′, tang. to the cyl. Give dist. of each trace from G. L.

327. Take BN, Fig. 12, as the axis of a cyl. of revolution with dia. 1¼″. Thru E, on the side furthest from V, pass a plane, sSs′, tang. to the cyl. Give dist. of each trace from G. L.

328. The pt. A, Fig. 12, is on the upper surface of the left hand cyl. Pass a plane tTt′ thru A and tang. to the cyl. Give ∠ KTt′.

329. If the elements of the cyl. K, in Fig. 12 are inclined 60° to H, and *a* is the H proj. of a point on the lower surface of the cyl., find *a*′, and pass a plane sSs′, thru A and tang. to the cyl. Give ∠ RSs′.

330. Thru C, Fig. 11, pass two planes, tTt′ and sSs′, tang. to the left hand cyl. Give ∠s RTt′ and RSs′.

331. Thru G, Fig. 11, pass two planes, sSs′ and tTt′, tang. to the rt. hand cyl. Give ∠s KS*s*′ and KT*t*′.

332. Thru M, Fig. 11, pass a plane, tTt′, tang. to the rt. hand cyl., on the side furthest from the V plane. Give ∠ bet. traces on drawing.

333. Thru D, Fig. 11, pass a plane, sSs′, tang. to the left hand cyl. on the side nearest the V plane. Give ∠ bet. traces on drawing.

334. Thru Q, Fig. 12, pass a plane sSs′ tang. to a cyl. of revolution whose axis is BN, and whose dia. is 1¼″. Give dist. bet. traces on drawing.

335. Thru A, Fig. 12, pass a plane, sSs′, tang. to a cyl. of revolution whose axis is BN, and whose dia. is 1″. Let the plane be passed on the *lower* side of the cyl. Give dist. bet. traces on drawing.

336. Thru D, Fig. 12, pass a plane, tTt′, tang. to a cyl. of revolution whose axis is BN, and whose dia. is 1″. Let the plane be passed on the *lower* side of the cyl. Give dist. bet. traces on drawing.

337. Let the ellipse in Fig. 12, be taken as the base of a cyl. whose right section is a circle and whose elements

run || to V. Const. the projs. of the cyl., and pass two planes, sSs′ and tTt′, tang. to it thru D. Give distances RS and RT.

338. Thru Q, Fig. 12, pass a plane, sSs′, tang. to the left hand cyl. on the side nearest the V plane. The elements of the cyl. make an ∠ of 60° with H. Give ∠ RSs′.

339. Pass a plane, tTt′, tang. to the left hand cyl. of Fig. 11 and || to the line CD. Give ∠ bet. traces on drawing.

340. Pass a plane, sSs′, tang. to the rt. hand cyl. of Fig. 11 and || to the line GM. Give ∠ bet. traces on drawing.

341. Pass a plane, sSs′, tang. to the left hand cyl. of Fig. 11, and || to the line DG. Give ∠ bet. traces on drawing.

342. Pass a plane, tTt′, tang. to the left hand cyl. of Fig. 11, and || to the line DM. Give ∠ bet. traces on drawing.

343. Pass a plane, sSs′, tang. to left hand cyl. of Fig. 12, and || to the line AB. The axis of the cyl. makes an ∠ of 60° with H.

344. Pass a plane, tTt′, tang. to the rt. hand cyl. of Fig. 11, and || to the G. L. Give dist. of Tt′ from the G. L.

345. Assume a rt. circular cyl. whose axis is BN, Fig. 12, and whose dia. is 1¼″. Pass a plane, sSs′, tang. to it and || to AB. Give dist. bet. traces on drawing.

346. Assume a rt. circular cyl. with axis BN, Fig. 12, and dia. 1¼″. Pass a plane, sSs′, tang. to it and || to OQ. Give dist. bet. traces on drawing.

347. Assume a rt. circular cyl. with axis BN, Fig. 12, and dia. 1″. Pass a plane sSs′ tang. to it and || to OB. Give dist. bet. traces on drawing.

348. Thru M, on the *lower* surface of the rt. hand cone in Fig. 12, pass a plane, sSs′, tang. to the surface. Give ∠ KSs′. Const. double size.

349. Take o', Fig. 12, as the V proj. of a pt. on the surface of the cone, and on the side toward V. Pass a plane, tTt′, thru this pt. tang. to the cone. Give ∠ RTt′. Const. double size. (No attention should be paid to the H proj. of O as given in the figure.)

350. Assuming Q, Fig. 12, as the vertex of a cone with the ellipse as base, pass a tang. plane, tTt′, thru the pt. (on the lower surface of the cone) of which *m* is the H proj. Give ∠ KTt′.

351. Thru E, Fig. 12, situated on the surface of the cone furthest from the V plane, pass a plane, tTt′, tang. to the cone. Give ∠ KTt′. Const. double size.

352. The vertex of a rt. circular cone is at B, Fig. 12, Axis is || to the G. L. Base is 2½″ to the rt. of the vertex, and 2″ in dia. Thru C, on the upper surface of the cone, pass a plane sSs′, tang. to the surface. Give ∠ bet. traces on drawing.

353. Take N, Fig. 12, as the vertex of a cone of revolution with axis || to the G. L. The base is 1¾″ in dia. and 2¼″ to the left of N. Thru E, on surface furthest from V plane, pass a plane, sSs′, tang. to the cone. Give ∠ bet. traces on drawing.

354. Take W, Fig. 12, as the vertex of a cone of revolution with axis coincident with the G. L. The base is 2½″ to the left of W, and 2¼″ in dia. Thru E, a pt. on the surface in 1st quadrant, pass a plane sSs′, tang. to the cone. Give ∠ bet. traces on drawnig.

355. Pass two planes, sSs′ and tTt′, thru O, Fig. 12, tang. to the surface of the left hand cone. Give ∠ˢ RTt′ and RSs′.

356. Thru Q, Fig. 12, pass a plane, tTt′, tang. to the surface of the rt. hand cone on the side furthest from the V plane. Give ∠ RTt′.

357. Thru B, Fig. 12, pass a plane, tTt′, tang. to the right hand side of the left hand cone. Give ∠ RTt′.

358. Assuming the rt. hand circle, Fig. 11, as the base

of a cone, and G as the vertex, pass a plane, tTt′, thru D, and tang. to the surface of the cone on its left side. Give ∠ KTt′.

359. Assume C, Fig. 11, as the vertex of a cone of revolution, with axis || to the G. L., base 1¼″ in dia., and 2½″ to the right of C. Pass two planes, sSs′ and tTt′, thru D and tang. to the surface. Give ∠s KSs′ and KTt′.

360. Assume D, Fig. 11, as the vertex of a cone of revolution with axis || to the G. L., base 1½″ in dia. and 2¾″ to the left of D. Pass two planes, sSs′ and tTt′, thru C and tang. to the surface. Give ∠s WSs′ and WTt′.

361. Assume R, Fig. 11, as the vertex of a cone of revolution with axis lying in G. L., base 1½″ in dia. and 3″ to the left of R. Pass a plane, tTt′, thru G and tang. to the upper surface of the cone. Give ∠ bet. traces on drawing.

362. Pass a plane, sSs′, tang. to the surface of the left hand cone, Fig. 12, on the side nearest V, and || to the line AB. Give ∠ RSs′.

363. Pass a plane, sSs′, tang. to the right hand cone in Fig. 12 and || to the line OQ, on the side furthest from V plane. Give ∠ RSs′. Const. double size.

364. Take the circle in Fig. 12, as the base of a cone, and A as the vertex, and pass a plane sSs′, tang. to the surface on the side furthest from V, and || to the line OQ. Give ∠ bet. G. L. and V trace. Const. double size.

365. Assume the rt. hand circle in Fig. 11 as the base of a cone, and G as the vertex. Pass two planes, sSs′, and tTt′, || to CD and tang. to the surface of the cone. Give ∠s KSs′ and KTt′.

366. Assume a rt. circular cone with vertex at D, Fig. 11, axis || to the G. L., base 1¼″ in dia. and 2½″ to the right of D. Pass two planes sSs′, and tTt′, tang. to the cone, and || to the line GM. Give ∠s RSs′ and RTt′.

367. Assume a rt. circular cone with vertex at B, Fig. 12, axis || to the G. L., base 1¼″ in dia. and 2½″ to the

left of B. Pass two planes, sSs′ and tTt′, tang. to the cone and || to the line OQ. Give ∠ˢ KSs′ and KTt′.

368. Assume a rt. circular cone, axis in the G. L. dia of base = 1½″, alt. = 3″, vertex on left side of base. Pass two planes, sSs′ and tTt′, tang. to the cone and || to AB, Fig. 12. Give ∠ bet. traces on drawing for each plane.

369. Pass a plane sSs′, tang. to the left hand cyl. in Fig. 11, and making an ∠ of 75° wtih H. Show element of contact and give ∠ bet. traces on drawing. (Four solutions.)

370. Pass four planes, rRr′, sSs′, tTt′ and uUu′, tang. to the left hand cone, Fig. 12, and making an ∠ of 60° with H.

371. The H proj. of a helix with vertical axis has a radius of 1.75″. Center of circle is 2.25″ from G. L. Curve rises 5″ for each revolution in a *clockwise* direction, and pierces H plane 2.25″ from G. L. on *right hand side* of axis. The H proj. of a pt. R on the surface of a helical convolute is assumed 2″ from the center of the H proj. and on a line thru the center bisecting the lower right hand quadrant. Pass a plane, sSs′, thru R and tang. to the surface. Give ∠ bet. G. L. and S*s*′.

372. The vertical axis of a helix is 2.5″ from V. Generating pt. moves *counter-clockwise* at a dist. of 2″ from the axis. Curve springs from H plane on *left* side of axis. Pitch of helix is 4″. The H proj. of a pt. R on the surface of a helical convolute is assumed 2.25″ from center of H proj. and on a line in the lower left hand quadrant that makes an ∠ of 30° with the G. L. Thru this pt. R pass a plane tTt′, tang. to the helical convolute. Give ∠ bet. G. L. and V trace.

373. The vertical axis of a helix is 2.5″ from V. Generating pt. moves *counter-clockwise* at a dist. of 2.2″ from the axis. Curve springs from H plane on *left* side of axis. Pitch of helix is 5 ″. The H proj. of a pt. R on the sur-

face of a helical convolute is 2.5″ from center of H proj and on a line in the lower left hand quadrant that passes thru the center and is inclined 30° to the G. L. Pass a plane, tTt′, thru R and tang. to the helical convolute. Give ∠ bet. G. L. and V trace.

374. The H proj. of a helix is a circle of 2″ radius. Center of circle is 2.5″ from G. L. Curve rises 3″ for each revolution, and springs from H plane at a pt. 2.5″ from G. L. on *right hand* side of axis. Generating pt. revolves *clockwise* as it rises. The H. proj. of a pt. R, on the surface of a helical convolute, is assumed 2.25″ from the center of the circle on a line thru the center in the lower right hand quadrant, inclined 30° to the G. L.

Thru R, pass a plane, sSs′, tang. to the helical convolute. Give ∠ bet. G. L. and V trace.

375. Pass a plane, uUu′, tang. to the helical convolute of Prob. 374 and || to the line DW, Fig. 7. Give (a) dist. from U to pt. where V proj. of axis intersects G. L. (b) acute ∠ bet. G. L. and V trace.

376. The H proj. of a helix has a radius of 2.5″. Center of circle is 3″ from G. L. Curve rises 4″ for each revolution in a *counter-clockwise* direction, and pierces H plane 3″ from G. L. on left side of axis. Assuming this helix as the directrix of a helical convolute, pass a plane, mMm′, || to the line GC, Fig. 8, and tang. to the helical convolute. Give (a) dist. from M to pt. where V. proj. of axis intersects G. L. (b) acute ∠ bet. G. L. and V trace.

377. Const. the projs. of a rt. circular cone resting on an element BE, Fig. 8, in H, vertex at B, dia. of base = 1¼″.

378. Thru the pt. C, Fig. 11, pass a line tang. to the two cyl's. Show the two pts. of contact, O and Q. There are four possible solutions. In this case let the line touch both cyls. on the side toward the 2nd quadrant.

379. Thru O, Fig. 8, pass a line OF that shall be ½″

from DG and ⅝″ from AB. There are four possible solutions. In this case let the required line pass on the side of the given lines toward the V plane.

PROBLEMS RELATING TO PLANES TANGENT TO WARPED SURFACES

380. Assume the hyperbolic paraboloid with directrices BD and EM, Fig. 8, and DE and BM as two directrices. Pass a plane, kKk′, tang. to the surface at a pt. F, on the surface, whose V proj. is ⅞″ above S. Give dist. SK. Const. double size.

381. Assume the hyperbolic paraboloid of Prob. 311. Pass a plane, kKk′, tang. to the surface at a pt. F, on the surface, whose H proj. is 1¼″ below S. Give dist. KS.

382. An hyperboloid of revolution of one nappe is generated by revolving DW, Fig. 8, about GE as an axis. Pass a plane, kKk′, tang. to the surface at a pt. F, on the surface, ¾″ from the axis and 1″ from V. Give ∠ bet. traces on drawing.

383. An hyperboloid of revolution of one nappe is generated by revolving EM, Fig. 8, about ON as an axis. Pass a plane, kKk′, tang. to the surface at a pt. F. on the surface, ¾″ from the axis and ⅝″ from V (above the gorge circle). Give ∠ bet. traces on drawing. Const. double size.

384. Pass a plane, tTt′, tang. to the hyperboloid of revolution in Prob. 382 at a pt. J, on the surface below the gorge circle ¾″ from the axis and ⅞″ from V. Give ∠ bet. traces on drawing. Const. double size.

385. Take the helix of Prob. 376 as the directrix of a helicoid, whose elements are inclined 45° to H. Assume a pt. M on an element whose H proj. bisects the lower left hand quadrant of the circle, and 2″ from the center.

(a) Const. V proj. of helix for a half revolution.

(b) Const. curve of intersection bet. H plane and helicoid for a quarter revolution.

(c) Pass a plane, sSs′, thru M, tang. to the helicoid, giving ∠ bet. G. L. and V trace.

386. Take the helix of Prob. 374 as the directrix of a helicoid whose elements are inclined 45° to H. Assume a pt. M, 1.5″ from the axis, and on an element whose H proj. is in the lower right hand quadrant of the circle and inclined 30° to the G. L.

(a) Const. V proj. of helix for a half revolution.

(b) Const. H trace of helicoid for a quarter revolution.

(c) Pass a plane, tTt′, thru M, tang. to the helicoid, giving ∠ bet. G. L. and V trace.

387. Take the helix of Prob. 373 as the directrix of a helicoid whose elements are inclined 45° to H. Assume a pt. M, 2″ from the axis, and on an element whose H proj. makes an ∠ of 30° to the G. L. and is in the lower left hand quadrant of the circle.

(a) Const. V proj. of helix for a half revolution.

(b) Const. curve of intersection bet. H plane and helicoid for a quarter revolution.

(c) Pass a plane, sSs′, thru M and tang. to the helicoid, giving ∠ bet. G. L. and V trace.

388. Assume GE, Fig. 8, as the axis of the helix of Prob. 300. Take this helix as the directrix of a helicoid whose elements are inclined 60° to H. Pass a plane, kKk′, tang. to the helicoid and ⊥ to DT.

389. The directrices of a conoid are: the circle of Fig. 12, lying in H, and the line BN (produced). The plane directer is || to the P plane. Pass a plane, kKk′, tang. to the conoid at a pt. on the surface ½″ above H and ⅞″ from V. Const. double size. Give ∠ bet. traces on drawing.

390. The directrices of a conoid are: the ellipse in Fig. 12, lying in H, and the line DN. The directer is a plane ⊥ to H with H trace passing thru Rn. Pass a plane,

kKk′, tang. to the conoid at a pt. on the surface ½″ above H and 1″ from V. Const. double size.

PROBLEMS RELATING TO PLANES TANGENT TO DOUBLE CURVED SURFACES

391. A sphere of 3″ dia. has its center at D, Fig. 12. Pass a plane, sSs′, tang. to the sphere thru the pt. E on the side nearest V. Give ∠ bet. traces on drawing.

392. A sphere of 2″ dia. has its center at D, Fig. 7. Pass a plane, kKk′, tang. to the sphere thru the pt. Q on the side nearest V. Give ∠ bet. traces on drawing.

393. A sphere of 3¼″ dia. has its center at D, Fig. 12. Pass a plane, sSs′, tang. to the sphere thru the pt. M on the lower side of the sphere. Give ∠ bet. traces on drawing.

394. A sphere of 2¼″ dia. has its center at C, Fig. 11. Pass a plane, sSs′, tang. to the sphere thru the pt. B on the upper surface of the sphere. Give dist. bet. traces on drawing.

395. A sphere of 2½″ dia. has its center in the G. L. at K, Fig. 11. Pass a plane, tTt′, tang. to the sphere at the pt. A (A being on the surface in the 1st quadrant). Give dist. KT.

396. A sphere of 3″ dia. has its center in the G. L. at R, Fig. 11. Pass a plane, tTt′, tang. to the sphere thru the pt. E on the surface (E being in the 1st quadrant). Give dist. RT.

397. An ellipse with axes 3″ and 2″ has its center at B, Fig. 13, and its major axis ⊥ to H. As it is revolved about its major axis it generates an ellipsoid of revolution.

Thru the pt. C, on the upper part of the surface, pass a plane, tTt′, tang. to the ellipsoid. Give dist. *a*′T.

398. Thru the pt. F, on the lower side of the ellipsoid of Prob. 397, pass a plane, tTt′, tang. to the surface. Give dist. *a*′T.

399. Thru the pt. E, on the upper surface of the ellipsoid of Prob. 397, pass a plane, tTt′, tang. to the surface. Give dist. *a*′T.

400. Pass planes, sSs′ and tTt′, tang. to the ellipsoid of Prob. 397 thru the two points on the surface whose V proj. is *d*′. Give dist. *a*′S and *a*′T.

401. A circle of 2″ dia. and || to V has its center at G, Fig. 13. As it is revolved about AB as an axis, it generates a *torus*.

Thru the pt. K, on side furthest from V plane, pass a plane, sSs′, tang. to the torus. Give ∠ bet. traces on drawing.

402. Thru the pt. F, on the upper side of the torus of Prob. 401, pass a plane, sSs′, tang. to the surface. Give ∠ bet. traces on drawing.

403. Thru the pt. C, on the lower side of the torus of Prob. 401, pass a plane, sSs′, tang. to the surface. Give ∠ bet. traces on drawing.

404. Assume a torus generated by revolving a circle of 1½″ dia. about the line BG, Fig. 13, as an axis; the center of the circle being ¾″ from the axis, and the pt. B being taken as the center of the torus.

Pass two planes, sSs′ and tTt′, tang. to the surface at the pts. whose H proj. is at *c*.

405. Assume a sphere of 2″ dia. with center at B, Fig. 13. Pass two planes, sSs′ and tTt′, thru the line MN and tang. to the sphere. Give ∠ˢ bet. traces on drawing.

406. Assume a sphere of 2″ dia. with center at A, Fig. 8. Pass two planes, sSs′ and tTt′, thru the line DG and tang. to the sphere. Give ∠ˢ bet. the traces on drawing.

407. Assume a sphere of 2½″ dia., center in G. L. at R, Fig. 12. Pass two planes, sSs′ and tTt′, thru the line OQ, tang. to the sphere. Give ∠ˢ bet. the traces on drawing.

408. Assume a sphere of 2″ dia., center in G. L. at *a*′,

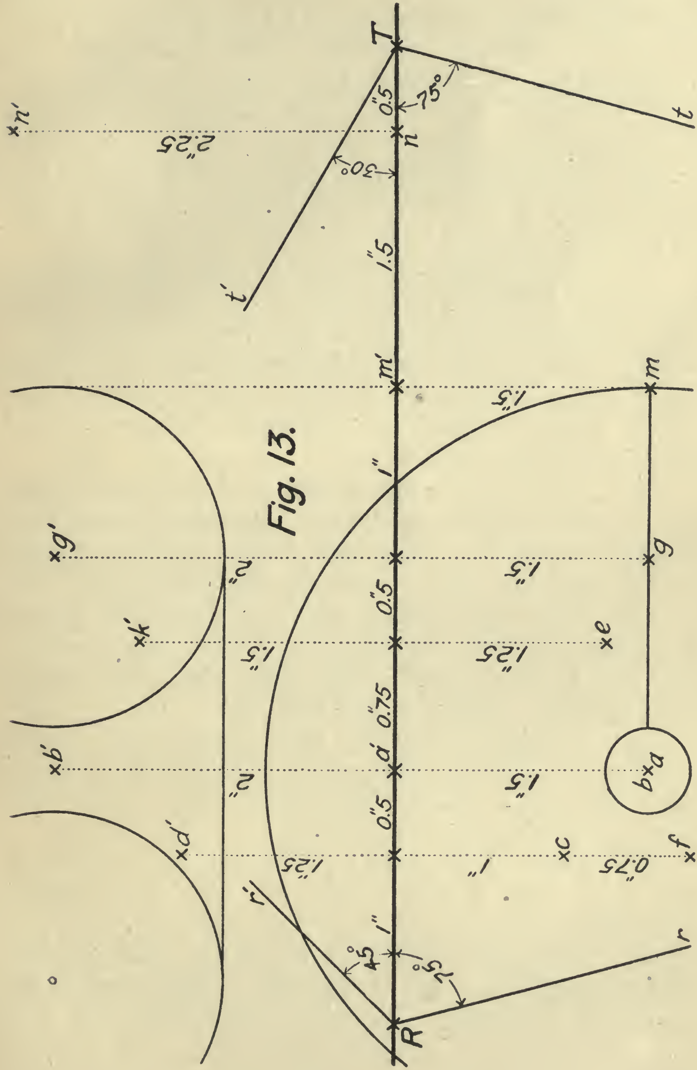


Fig. 13.

Fig. 13. Pass two planes, sSs′ and tTt′, thru the line MN, tang. to the sphere. Give ∠ˢ bet. traces on drawing.

409. Thru the line MD, Fig. 4, pass two planes, sSs′ and tTt′, tang. to a sphere of 2″ dia. and center in G. L. at G. Give ∠ˢ bet. traces on drawing.

410. Pass two planes, sSs′ and tTt′, thru the line BD, Fig. 8, and at a dist. of 1″ from the pt. M. Give ∠ˢ bet. traces on drawing.

411. Pass a plane, sSs′, thru the line MN, Fig. 13, tang. to the ellipsoid of Prob. 397 on its upper side. Show pt. of tangency, O, and give ∠ bet. traces on drawing.

412. Pass a plane, sSs′, thru the line MN, Fig. 13, and tang. to the torus of Prob. 401 on the upper portion of the interior surface. Show pt. of tangency, O, and give ∠ bet. traces on drawing.

413. Three equal spheres of dia. 1½″ have their centers at B, D and O respectively, Fig. 8. Pass eight planes tang. to all three spheres. Letter them from S to Z in the order of their distances from *b′* measured along the G. L.

414. Two equal spheres of dia. 1″ have their centers at D and O respectively, Fig. 8. A third sphere of dia. 1½″ has its center at B. Pass eight planes tang. to all three spheres. Letter as in Prob. 413.

415. A sphere of ¾″ dia. has its center at G, Fig. 8. A sphere of 1½″ dia. has its center at D, and one of 1″ dia. has center at O. Pass eight planes tang. to all three spheres. Letter as in Prob. 413.

416. Const. a plane, kKk′, that shall be ½″ from B, Fig. 8, ⅝″ from A, and 1″ from D. How many possible solutions?

417. Given the ellipsoid of Prob. 397, const. the *tangent cone* with M, Fig. 13, as vertex. Show both projs. of the line of contact.

INTERSECTION BETWEEN CURVED SURFACES AND LINES

418. Find the pts., O and Q, where the line MN, Fig. 11, pierces the right-hand cyl.

419. Find the pts., F and S, where the line OQ, Fig. 12, pierces the cyl. of revolution whose axis lies in BN, and whose dia. is 1¼″.

420. Find the pts., O and Q, where a line ‖ to the G. L. thru C, Fig. 11, pierces the right hand cyl.

421. With *d* and *d′* as centers, Fig. 11, and a 2″ radius, strike the lower left hand quadrants of two circles. These are the H and V projs. of a curved line. Find the pts., O and Q, where it pierces the left hand cyl.

422. Find the pts., F and S, where the line AB, Fig. 12, pierces the cone whose vertex is Q.

423. Find the pts., F and S, where a line ‖ to the G. L., ¾″ above H and 1½″ in front of V, pierces the left hand cone in Fig. 12.

424. With *q*, Fig. 12, as center and 1½″ radius, describ the upper left hand quadrant of a circle. With *q′* as center, and same radius, describe the lower left hand quadrant of a circle. These are the H and V projs. of a curved line. Determine whether this curve intersects the cone. If so, find the piercing pts., F and S. Draw double size.

425. Assume a sphere of 2″ dia. and center at O, Fig. 8. Find the pts., F and K, where the line GM pierces the sphere.

426. Assume a sphere of 2½″ dia. and center at O, Fig. 8. Assume also a helix with axis GE, dia. of H proj. 2½″, and pitch 3″. Find all the pts., A, B, C, etc., where the helix pierces the sphere.

INTERSECTION BETWEEN CURVED SURFACES AND PLANES—DEVELOPMENT OF SURFACES

427. Assume a right circular cyl. of 4″ dia., base in H, axis ⊥ to H, and center of base 2¼″ from G. L. The cyl. is cut by a plane, tTt′; the pt. T being 3″ to the right of the axis, the H trace inclined 75°, and the V trace 35° to the G. L. (Tang. 35°=.7002.)

Show the projs. of the line of intersection. Show highest pt. of curve, E, and lowest pt., F. Show pts. of tangency, J and K, to left and right hand elements, and find the true form of the curve. Show all construction lines necessary to obtain these various points.

Develop that part of the cylindrical surface included bet. the H plane and the plane tTt′. Put *longest element in the middle.*

428. A right circular cyl. of 3½″ dia. is cut by a plane, tTt′, inclined 60° to the axis, and intersecting the plane of the base in a line 2¾″ from the axis.

Show the true size and shape of the curve cut from the plane, tTt′, and develop the portion of the cylindrical surface included bet. the base and the plane tTt′. Make pattern with *longest element in middle.*

429. Make a pattern for the section B in the stovepipe elbow shown in Fig. 14, half size.

430. Assume AB, Fig. 8, as the axis of an oblique cyl. with circular base in H. Dia. of base 1¼″. The cyl. is cut by the plane rRr′. Show (a) the projs. of the curve of intersection, (b) the pts. of tangency, J and K, to the outside elements in the V proj., (c) a tang. at some pt. on the curve, (d) the true shape of the curve. Show all necessary construction lines. Const. *triple size.*

431. Assume ED, Fig. 8, as the axis of an oblique cyl. with circular base in H. Dia. of base, 1″. The cyl. is cut by a plane, sSs′. Show the projs. of the curve of intersection. Show also a *right section* made by a plane thru the pt. S in the G. L., and make a pattern for that part

of the cylindrical surface lying bet. these two cutting planes. Put longest element in middle of pattern. Const. *triple size.*

432. Thru W, Fig. 11, pass a plane, wWw′, ⊥ to the elements of the left hand cyl. Show the projs. of the right section thus cut from the cyl. Show the pts. of tangency, J and K, to the right and left hand elements. Draw a tang. at some convenient pt. on the curve, and develop that portion of the cylindrical surface lying bet. wWw′ and the H plane. Put longest element in middle. Const. *triple size.*

433. Given a cyl. with circular rt. section, radius 1″; axis ‖ to V, 2¼″ from V, in 1st quadrant, and inclined 60° to H. The cyl. is cut by two planes, sSs′ and tTt′, ‖ to the G. L. S*s* is 3⅝″ below the G. L. T*t′* is 4½″ above the G. L. The two planes intersect in a line AB in 1st quadrant, 2¼″ from H, and 3″ from V. Show projs. of curve of intersection bet. planes and cyl. Develop that portion of cylindrical surface lying bet. H plane and the cutting planes. Put highest element in middle.

434. A rt. circular cone, radius of base = 2.25″, alt. = 4.5″, is cut by a plane which is inclined 60° to the axis, and cuts the axis in a pt. 2.75″ from the vertex. (a) Show top view of curve of intersection. (b) Show the curve of intersection in its true dimensions, giving lengths of the major and minor axes of the ellipse. (c) Develop the surface of the cone with the curve of intersection; highest element of frustum in the middle. Give the ∠ of the circular sector.

435. A rt. circular cone, radius of base = 2″, alt. = 4″, is cut by a plane inclined 67½° to the axis, and cutting the axis in a pt. 2¾″ from the vertex. (a) Show top view of curve of intersection. (b) Show the curve in its true dimensions, giving lengths of major and minor axes of ellipse. (c) Develop the surface of the cone with the curve of intersection, putting highest part of curve in the middle. Give the ∠ of the circular sector.

436. A cone having a vertex ∠ of 90°, and alt. 2½″,

has a parabola cut from it by a plane whose shortest dist. from the vertex of the cone is ¾″. Show the H proj. of the section and its true form. Make a pattern for that part of the cone *below* the cutting plane.

437. A cone having a vertex ∠ of 60°, and alt. 3″, is cut by a plane ‖ to the axis and ½″ from it. Show the true form of the section and develop the surface of the cone, with the curve of intersection.

438. Assume A, Fig. 8, as the vertex of an oblique cone, and B the center of its circular base lying in H. Dia. of base = 1¼″. The cone is cut by the plane rRr′. Show the projs. of the curve of intersection, the pts. of tangency, J and K, to the V projs. of the left and right hand elements, and a tang. to the curve at a convenient pt. Show the true shape of the curve. Develop that part of the conical surface *above* the cutting plane, putting longest element in middle. Make drawings *triple size.*

439. Assume D, Fig. 8, as the vertex of an oblique cone, and B the center of its circular base in H. Dia. of base = 1½″. The cone is cut by the plane rRr′. Show projs. of the curve of intersection, pts. of tangency, J and K, to outside elements in V proj., and a tang. to the curve at a convenient pt. Show the true shape of the curve. Develop that part of the conical surface above the cutting plane, starting with the extreme right hand element. Make drawings *double size.*

440. Find the curve of intersection bet. the rt. hand cone in Fig. 12, and the plane tTt′. Show pts. of tangency, J and K, with outside elements in V proj., and a tang. to the curve at some convenient pt. Show the true shape of the curve. Make a pattern for the frustum bet. base and cutting plane, placing the extreme rt. hand element in the middle. Const. *double size.*

441. Find the intersection bet. the rt. hand cyl., Fig. 11, and a prism with base *ned*, in H, edges ‖ to V and inclined 30° to H, toward the right. Develop that portion

of cylindrical surface lying bet. H, and a plane || to H, $1\frac{1}{2}''$ above it. Const. *double size.*

442. Assume a rt. handed helix with axis ON, Fig. 8. Pitch $3''$; curve springs from the pt. E. With this as directrix, assume a helical convolute, and const. the projs. of the curve of intersection cut by the plane sSs′ from both upper and lower nappes.

443. Develop the surface of the lower nappe of the above helical convolute.

444. Find the intersection of the helical convolute of Prob. 442 with the plane tTt′.

445. Develop the surface of the helical convolute of Prob. 444, bet. the H plane and the plane tTt′.

446. An ellipsoid of revolution, $3''\text{x}1\frac{1}{2}''$, has its center at N, Fig. 8; apex at O; longer axis $\perp$ to H. Find the intersection with the plane tTt′. Show highest and lowest pts. E and F. Draw a tang. at some convenient pt. and show the true shape of the curve. Const. *double size.*

447. Find the intersection bet. the plane tTt′, Fig. 13, and the torus of Prob. 401. Const. a tang. at some convenient pt. Show the highest and lowest pts. E and F, and the true shape of the curve.

448. Find the intersection of the plane, rRr′, Fig. 13, with the torus of Prob. 401. Const. a tang. to the curve at some convenient pt. Show the highest and lowest pts. E and F, and the true shape of the curve.

449. Assume CD and MN, Fig. 8, as the directrices, and the H plane as the directer, of an hyperbolic paraboloid. Find the projs. of the curve cut from this surface by rRr′, and show its true shape.

450. Find the curve of intersection bet. the hyperbolic paraboloid of the above problem and the V plane of proj.

451. Assume CD and MN, Fig. 8, as the directrices of an hyperbolic paraboloid with the V plane as directer. Show the projs. of the curve of intersection bet. this surface and the plane rRr′, and the true form of the curve.

452. The directrices of a conoid are: a circle in H with center at N, Fig. 8, dia. 1½″, and a rt. line thru O, || to the G. L. The plane directer is the P plane. Find the intersection of the conoid with the plane tTt′, and show the true form of the curve. Const. *double size.*

453. A shaft running east and west on the second story of a building is connected by a belt with a shaft running north and south on the first story. The first shaft is 8 ft. above the floor, and the dist. bet. the shafts is 10 ft. The width of the belt is 1 ft. and the dia. of the pulleys is 2 ft. It is desired to cut an opening in the floor 2″ wide of the proper form and in the proper place to allow the belt to pass thru. Const. to scale 1½″=1′.

454. Assume ON, Fig. 8, as the axis of a right handed helix, dia. of H proj. = 2″, pitch = 2½″. It is the directrix of a helicoid whose elements are inclined 30° to the axis. Show the projs. of the intersection with the plane sSs′, const. a tang. at a convenient pt. on the curve, and show the true form of the curve. Const. *double size.*

INTERSECTION OF CURVED SURFACES

455. The axes of a rt. circular cyl. and a rt. circular cone are || and 1¼″ apart. Dia. of cyl. = 1½″. Alt. of cyl. = 3¼″. Dia. of base of cone = 4¼″; alt. of cone 3½″. Bases of both figures rest in H, their centers being located on a line inclined 30° to the G. L. Show projs. of the line of intersection. Make pattern for conical surface. Make pattern for projecting portion of cyl.

456. A sheet iron cylindrical tank whose dia. is 4 ft. and alt. 4 ft. has a conical top. Height of cone = 2 ft. A pipe of 1½ ft. dia. enters from above. Its axis passes thru junction of cone and cyl. and makes an ∠ of 30° with the axis of the cyl. Show projs. of intersection, and cut patterns for pipe, top of tank and cyl. Make longest element on pipe = 3 ft. Const. to scale 1″=1′.

457. Show projs. of intersection bet. cone and cyl. in Fig. 15. Const. a tang. at some convenient pt. on curve.

458. Find the intersection of the cyl. with circular base, Fig. 16, and the cone with elliptical base (axes 2 5/16″ and 1⅞″). Draw a tang. at some pt. on the curve.

459. Make a pattern for spout of tea pot in Fig. 17. Show necessary construction work on paper.

460. Two rt. circular cyls. of dia. 3″ have their axes ½″ apart. They are each || to V, and the ∠ bet. the V projs. of their axes is 60°. Find the curve of intersection bet. the two cyls. and develop the surface of one of them. Place them in the most convenient position with respect to H.

461. A vertical tube of 5″ outside dia., 3″ inside dia., has a horizontal cylindrical hole bored thru it of 3″ dia. Dist. bet. axis of hole and axis of cyl. = ½″. Show projs. of outlines of hole upon a vertical plane to which the axis of the horizontal hole is inclined at 45°. Cut pattern for outside surface of tube, and show all necessary construction lines.

462. Let the elliptical base of the cone in Fig. 16, be taken as the base of a cyl. with axis || to the axis of the cone. Find the intersection bet. the two cylinders and draw a tang. at some pt. on the curve.

463. A cone of revolution rests with its base in H. Dia. of base = 4″; alt. = 4½″. Another cone of revolution has its axis inclined 45° to the axis of the first cone, intersecting it at a pt. ⅜″ above center of base of first cone. Dist. from this pt. to apex of second cone = 7″. ∠ at vertex of second cone = 30°. Show projs. of curve of intersection. Develop each cone.

464. Find the intersection bet. the two cones in Fig. 18, and const. a tang. at some pt. on the curve. Develop the surface of rt. hand cone showing the curve of base and the two curves of intersection. Const. to scale 1½″=1″.

465. Assume a cone of revolution with alt. 5½″ and dia. of base 3½″. Its axis is coincident with that of the

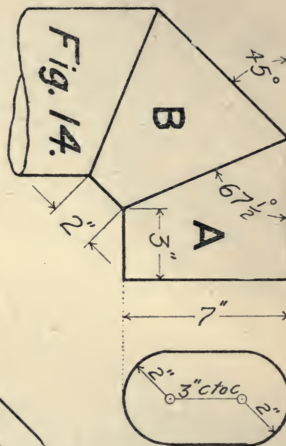


Fig. 14.

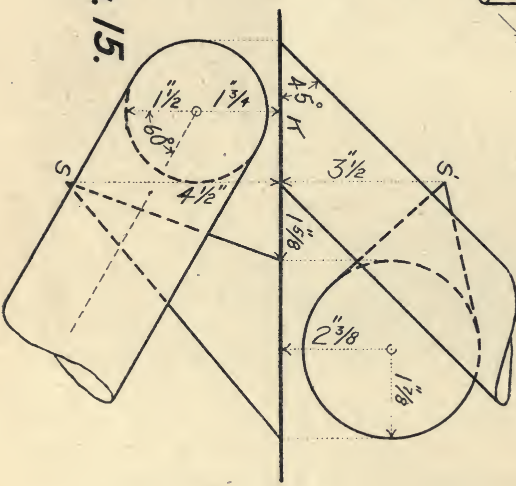


Fig. 15.

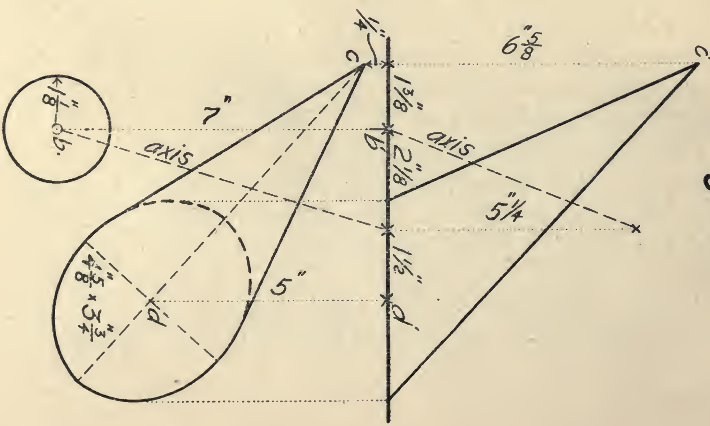


Fig. 16.

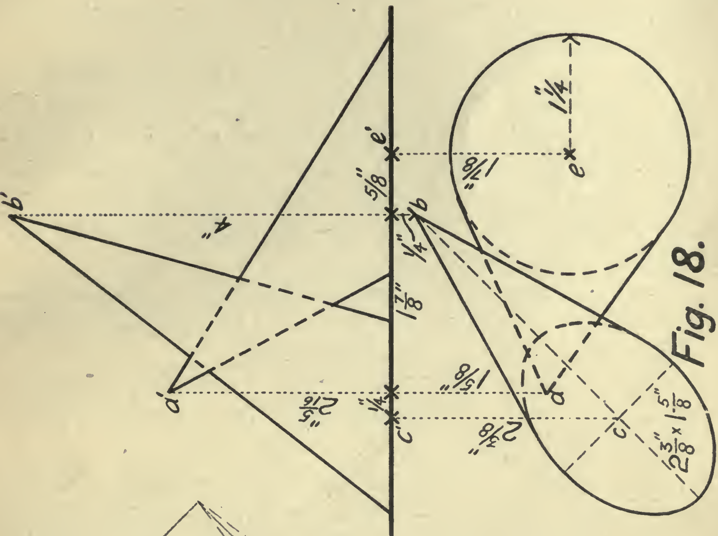

b'
4"
e'
5/8"
b
1/4"
$1\frac{7}{8}$"
a'
$2\frac{5}{16}$"
1/4"
c'
$1\frac{1}{4}$"
e
$1\frac{1}{8}$"
$1\frac{5}{8}$"
a
$2\frac{3}{4}$"
c
$2\frac{3}{8}$" x $1\frac{5}{8}$"

Fig. 18.

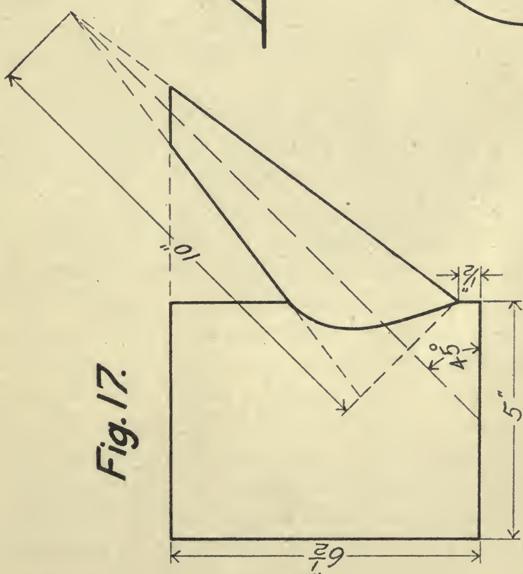

10"
1/2"
45°
5"
$6\frac{1}{2}$"

Fig. 17.

helicoid of Prob. 308. Find the curve of intersection bet. the two surfaces. Develop surface of cone.

466. Assume a cyl. of revolution, dia. 1½″, axis in H, and running thru the pts. *e* and *n*′, Fig. 8. It is cut by the hyperbolic paraboloid whose directrices are MN and WD and whose plane directer is || to H; and by a sphere with dia. 2″ and center in H at *d*. Find curves of intersection and develop surface of cyl. bet. hyperbolic paraboloid and sphere. Const. *double size.*

467. Assume a cyl. with axis ED, Fig. 8, and circular base in H; dia. of base 2″. It is cut by a sphere of 3″ dia., center at N. Find curve of intersection.

468. The left hand cone of Fig. 12 is cut by a sphere whose center is at B, and whose dia. is 2½″. Show projs. of curve of intersection. Const. *double size.*

469. The left hand cone of Fig. 12 is cut by a sphere whose center is at Q and whose dia. is 3¼″. Find curve of intersection and develop surface of cone. Const. *double size.*

470. Find the intersection bet. the torus of Prob. 401 and an ellipsoid of revolution generated by an ellipse revolving about its minor axis GM; the major axis being equal to 3″.

471. Find the intersection bet. the torus of Prob. 401 and a cone of revolution with center of base at M, vertex at B, and dia. of base = 3″.

472. An ellipse with axes 2″ and 3″ spins about its minor axis, AB, Fig. 13, and generates an *ellipsoid of revolution.* Find its intersection with a cone of revolution with vertex at B, center of base at M, and dia. of base = 3″.

473. A torus is generated by revolving a circle of 2″ dia. about a line in its plane 2¼″ from its center. Let this torus rest in a horizontal plane ¼″ above H and 2″ from V plane, with axis vertical. Find its intersection with a cone of revolution with base in H, axis 2″ from axis of

torus, dia. of base = $3\frac{1}{2}''$, alt. = $4''$. Take the H projs. of the axes on a line inclined $45°$ to the G. L.

474. An ellipsoid of revolution is generated by revolving a $2''$x$3\frac{1}{2}''$ ellipse about its minor axis AB, Fig. 13. Find its intersection with a cone of revolution with base in H, axis passing thru f, dia. of base = $3''$, alt. = $3\frac{1}{2}''$.

475. An ellipsoid of revolution is generated by revolving a $2''$x$3''$ ellipse, center at O, Fig. 8, about its major axis $\perp$ to H. Find its intersection with an oblique cone with circular base in H. Center of base at E, dia. of base = $2''$, vertex at M.

MISCELLANEOUS

476. Thru the pt. B, Fig. 8, pass a line, BK, that shall be $1''$ from G and $1\frac{1}{4}''$ from D. (Two solutions.)

477. Thru B, Fig. 8, pass a line, BK, that shall touch the line DG and be $1\frac{1}{2}''$ from N. (Two solutions.)

478. Thru D, Fig. 8, pass a line, DK, that shall be $1''$ from O, and $\frac{1}{2}''$ from NM. (Four solutions.)

479. Assume two horizontal lines, AB and CD, $2''$ apart. Their H projs. make an $\angle$ of $60°$. With these lines as axes, const. the projs. of two hyperboloids of revolution of one nappe so that they will roll upon one another transmitting motion with a velocity ratio of 2 : 3.

480. Same as above, the velocity ratio being 3 : 4.

481. A staircase vestibule is 10 ft. square. Dist. bet. first and second floors is 12 ft. A spiral staircase begins at the middle of one side, turns thru $270°$ and ends on the second floor. Each stair is $6''$ high, the well-hole is 4 ft. in dia. Show the projs. of the staircase. Scale, $\frac{1}{2}''=1'$.

482. The pts. O, B and D, Fig. 8, are the terminations of three concurrent edges of a rectangular parallelopiped. Const. its projs.

ANSWERS AND OTHER DATA

Problems in descriptive geometry, when worked by students on plates of uniform size should be so located that they will not interfere with one another, nor extend beyond the limits of the sheet. To facilitate the proper placing of the problems, we give here, in addition to the answer for each problem, the size of the rectangle in which the problem may be worked, and the coördinates of some known point within the rectangle.

To illustrate the use of these data, let us suppose that the paper on which the problems are to be worked is of the size 12″x16″, and that it is desired to have the student solve Problems 46, 55, 62 and 69 on that sheet. Refering to Prob. 46 in the following pages, it is seen that it may be worked within a space 6″x4½″ (the first dimension being the horizontal one), the point G being located 2⅝″ from the left edge and 2⅜″ from the lower edge. Now let a 6″x4½″ rectangle be cut from a sheet of paper and the point G located thereon by means of its coördinates. Similarly for Problems 55, 62 and 69. These pieces of paper may then be placed upon the 12″x16″ sheet in the most convenient positions, and the coördinates of the points G, S, etc., measured, taking the lower left hand corner of the plate as origin. The specifications may then be placed upon the blackboard in the following brief manner:

Plate 4

Prob. 46. Put G at (3½″, 9¼″).
Prob. 55. Put G at (8½″, 9¾″).
Prob. 62. Put *n′* at (1″, 2¾″).
Prob. 69. Put S at (12¾″, 4″).

If the answer to a problem is a linear measurement and

it has been constructed *"double size,"* the measurement should be divided by two.

Problem.

13.—2¼x4¾; K at (1⅛, 3⅞).
14.—4x5¼; G at (⅛, 2⅝).
15.—3½x5¾; *d* at (¾, ¾).
16.—5½x5½; S at (3⅞, 3⅛).
17.—2⅜x3¾; R at (⅛, 1⅞).
18.—4⅝x4⅝; K at (3, 1⅝).
19.—5¾x5¾; S at (3⅛, 2⅝).
20.—2½x3½; R at (1⅞, 1⅝).
21.—1½x1¼; S at (1⅜, ⅛).
22.—1½x1¾; S at (1⅜, 1⅝).
23.—1¼x2¼; K at (⅛, ⅝).
24.—1¾x1½; G at (⅛, ⅝).
25.—3⅜x4⅛; G at (½, 2). Ans. 2.80″.
26.—2⅛x5; G at (⅛, 2⅜). Ans. 1.87″.
27.—3⅛x3⅛; K at (⅞, 1⅞). Ans. 2.51″.
28.—3⅜x3⅝; S at (1, 1⅞). Ans. 3.37″.
29.—4⅛x3⅞; R at (¾, 2). Ans. 3.76″.
30.—4¼x4¼; S at (1⅛, 3⅛). Ans. 1.03″.
31.—3½x3⅝; G at (⅜, 1¾). Ans. 1.93″.
32.—2¾x3¼; K at (¾, 2). Ans. 1.97″.
33.—3½x3⅞; S at (1⅛, 2¼). Ans. 0.94″.
34.—2⅞x5; K at (1⅛, 3¾). Ans. 11°29′ with H, 63°35′ with V.
35.—4¼x4⅛; G at (⅜, 2⅜). Ans. 20°58′ with H, 15°34′ with V.
36.—4¾x3¾; S at (1, 1⅞). Ans. 21°45′ with H, 54°37′ with V.
37.—1¾x2¼; *d* at (⅛, ⅛). Ans. 0.82″.
38.—3½x4⅜; R at (1⅛, 1½). Ans. 0.88″.
39.—4¾x4¾; *d* at (⅛, ⅛). Ans. 0.81″.
40.—4¾x4¾; *d* at (⅛, ⅛). Ans. 0.61″.
41.—3¼x5; G at (⅝, 3⅞). Ans. 61°35′ or its supplement.

42.—3¾x4½; R at (⅛, 2⅝). Ans. 51° or its sup.
43.—4x4¾; S at (3⅞, 3⅛). Ans. 80°5′ or sup.
44.—6⅜x6⅜; G at (2¾, 1⅝). Ans. 19°30′ or sup.
45.—3¼x4⅝; G at (1⅝, 1⅝). Ans. 0°.
46.—6x4¼; G at (2⅝, 2⅜). Ans. 59°10′ or sup.
47.—4x4⅜; S at (3⅞, 3⅛). Ans. 51°0′ or sup.
48.—4⅜x3¾; K at (1½, 1⅞). Ans. 73°44′ or sup.
49.—5⅛x3⅞; R at (1⅝, 2¼). Ans. 83°20′ or sup.
50.—2¼x5¼; K at (1⅛, 3⅝). Ans. 39°45′ or sup.
51.—4½x3; S at (2⅞, 1⅝). Ans. 3°45′ or sup.
52.—3¼x4¾; G at (⅛, 1⅝). Ans. 3.37″.
53.—4x2¾; R at (⅛, 1⅝). Ans. 0.15″.
54.—2¾x4¾; G at (⅛, 3⅞). Ans. 56°.
55.—6½x3; G at (⅛, 1¾). Ans. 78°30′.
56.—5¾x5¾; S at (2⅝, 1⅝). Ans. 88°40′.
57.—6x6½; S at (2⅝, 3⅜). Ans. 51°25′.
58.—3¼x6¾; G at (⅛, 2⅞). Ans. 36°42′.
59.—4¾x6; R at (⅛, 3⅛). Ans. 132°53′.
60.—4x2¾; R at (⅛, 1⅝). Ans. 79°10′.
61.—4x5¼; G at (¼, 2¾). Ans. 74°30′.
62.—5¼x5¾; n' at (⅛, 2⅛). Ans. 29°20′.
63.—3x3½; R at (¼, 2⅜). Ans. 52°15′.
64.—2½x5; U at (⅛, 1⅜). Ans. 64°21′.
65.—2¾x6; G at (1¾, 3). Ans. 82°34′.
66.—4⅜x3¾; K at (1½, 1⅞).
67.—5⅞x5⅞; S at (2⅝, 1⅝).
68.—8⅝x5⅝; S at (½, 3⅝). Ans. $o = 1.95''$; $g = 2.30''$; $d = 1.44''$.
69.—8¼x5¾; S at (5½, 3). Ans. $a = 1.95''$; $d = 3.35''$; $g = 2.14''$.
70.—5x3¾; R at (1⅝, 2¼). Ans. 11°10′.
71.—5¾x4½; G at (⅛, 3¼). Ans. 10°10′.
72.—7¾x6¼; R at (⅛, 3⅜). Ans. 49°40′.
73.—6x6½; S at (2⅝, 3⅜). Ans. 53°45′.
74.—3¼x6¾; G at (⅛, 2⅞). Ans. 16°15′.
75.—4¾x6; R at (⅛, 3⅛). Ans. 64°30′.

76.—2x2¾; G at (⅛, 1⅜). Ans. 32°15′.
77.—2¾x4½; G at (⅛, 1⅜). Ans. 33°20′.
78.—5⅞x4⅛; G at (2⅝, 2⅜). Ans. 46°15′ or 149°15′.
79.—1⅞x3⅛; G at (⅛, 1¼). Ans. 130°45′.
80.—7x7¼; K at (1⅛, 3⅛). Ans. 0.10″.
81.—3⅞x4⅞; R at (⅛, 2¾). Ans. 1.26″.
82.—5¼x6¾; G at (⅛, 3⅛). Ans. 1.05″.
83.—4x2⅜; R at (2⅝, 1½). Ans. 0.19″.
84.—2½x1⅝; U at (⅛, 1⅜). Ans. 1.31″.
85.—4¾x2¼; K at (3⅛, ⅝). Ans. 1.69″.
86.—2⅛x3⅛; *d* at (⅞, ⅞). Ans. 1.02″.
87.—1¾x3; W at (1⅝, 1⅜).
88.—7x5½; K at (1½, 2). Ans. 1.00″.
89.—3⅛x4⅝; R at (⅛, 2½). Ans. 2.19″.
90.—6¼x5½; R at (5¼, 3¼). Ans. 0.23″.
91.—6x6⅝; G at (⅛, 3⅛). Ans. 0.62″.
92.—4⅛x4⅝; R at (⅜, 1⅝). Ans. 0.49″.
93.—1⅜x3; R at (1¼, 2). Ans. 1.89″ below G. L.
94.—2x3½; *q′* at (1, 1¾). Ans. 0.15″.
95.—2x11; R at (1⅞, 6⅜).
96.—1⅞x3⅜; *q′* at (¾, 1⅞).
97.—2¾x2¾; R at (1⅝, 1⅛).
98.—5x4; G at (⅛, 2⅝).
99.—7x8½; S at (4¾, 2¼).
100.—3¾x4¼; U at (¼, 2¾).
101.—3x3¾; R at (1⅝, 1⅛). Ans. 0.65″.
102.—6x5¾; S at (3⅞, 3⅛). Ans. 0.24″.
103.—3¾x4¾; K at (2⅞, 2⅝). Ans. 1.52″.
104.—2½x3; U at (⅛, 1⅝). Ans. 0.62″.
105.—3x5; S at (2⅛, 1⅝). Ans. 1.05″.
106.—3¾x3¾; U at (2¾, 1¾). Ans. 1.96″.
107.—3x5½; *d* at (¼, ¼). Ans. 2.06″.
108.—2½x2½; *n* at (1¼, 1). Ans. 0.56″.
109.—1¾x3; *d* at (⅛, ⅛). Ans. 1.37″.
110.—1x1½; W at (¾, ⅜). Ans. 0.5″.
111.— Ans. 1.73″.

112.— Ans. 1.15″.
113.—4x6; U at (1/8, 2 1/4).
114.—5 1/2x4; R at (1, 2 1/2).
115.—2 1/2x3 3/4; *d* at (1/8, 1 3/8).
116.—5 1/2x4 1/2; W at (3, 2 1/2).
117.—4 3/4x9 1/4; S at (1 5/8, 5 1/2). Ans. 39°40′.
118.—7 3/4x11 3/8; S at (2 1/8, 5 3/4). Ans. 158°15′.
119.—4x3 1/2; K at (1/8, 2). Ans. 0.45″.
120.—4x4 3/8; K at (1 1/2, 2 3/4). Ans. 1.28″.
121.—4 3/4x3 1/4; K at (1/8, 1 5/8). Ans. 4.49″.
122.—3 1/8x3 3/4; G at (1/8, 1 3/4). Ans. 0.17″.
123.—2 3/4x4 1/4; S at (5/8, 2 1/8). Ans. 1.5″.
124.—3 3/4x3 1/2; G at (1/8, 1 3/8). Ans. 3″.
125.—3x5 1/4; *c′* at (0, 3 1/4). Ans. 4.06″.
126.—1 5/8x2 5/8; R at (2, 2 1/8). Ans. 0.56″.
127.—4x5; K at (1 1/8, 2 5/8). Ans. Maj. 2.25″; Min. 0.41″.
128.—7 7/8x4; G at (1/8, 2 3/8). Ans. 7.60″.
129.—6 3/4x5 3/4; T at (3 7/8, 4). Ans. 0.98″.
130.—3 7/8x6 1/4; S at (1/2, 4 1/8). Ans. 3.14″.
131.—4 3/4x4 3/8; G at (1/8, 3). Ans. 2.11″.
132.—3 1/2x4 1/4; R at (5/8, 3 1/8). Ans. 1.5″.
133.—4x3 3/4; K at (1 1/2, 2). Ans. 0.43″.
134.—4x7 1/4; S at (2 3/8, 5 5/8). Ans. 0.33″.
135.—4 1/4x3 1/4; R at (2 3/8, 1 3/4). Ans. 2.30″.
136.—3 1/4x3 1/4; U at (1/8, 1 1/4). Ans. 0.99″.
137.—4x4; K at (1/8, 1 5/8). Ans. 1.38″.
138.—6x5; R at (3, 2 5/8). Ans. 2.83″.
139.—6 1/2x5; U at (3 1/4, 2 3/8). Ans. 3.12″.
140.—5 1/2x4 1/2; K at (1 5/8, 1 3/4). Ans. 1.54″.
141.—6 3/8x5 3/4; K at (4 1/8, 4 1/2). Ans. 0.17″.
142.—10x8 1/4; S at (5 1/4, 6 1/2). Ans. 3.39″.
143.—2 3/4x4 3/4; K at (1 5/8, 3 7/8). Ans. 2.92″.
144.—4 1/2x5; G at (1/8, 2 5/8). Ans. 0.14″.
145.—4x7; K at (2 7/8, 1 1/2). Ans. 0.71″.
146.—4 1/4x8; K at (1 5/8, 3 1/2). Ans. 0.56″.
147.—3 1/4x5 3/4; K at (1 1/8, 3 3/4). Ans. 0.91″.

148.—7x6½; S at (3⅜, 3). Ans. 2.03″.
149.—5½x6; b' at (3¾, 3⅜). Ans. 2.01″.
150.—1¼x2¾; K at (⅛, 1⅛). Ans. 0.96″.
151.—4¼x6½; G at (1⅝, 3⅜). Ans. 2.60″.
152.—1¾x3½; G at (⅛, 1⅜). Ans. 1.12″.
153.—1¾x 2¾; G at (⅛, 1⅜). Ans. 1.80″.
154.—4x5; S at (1, 1½). Ans. 0.99″.
155.—4x4⅝; U at (⅛, 1¾). Ans. 43°10′.
156.—5¾x6¼; S at (⅛, 4¼). Ans. 74°10′.
157.—5½x6; d at (¾, ¾). Ans. 12°.
158.—4x3; W at (1⅝, 1⅜). Ans. 38°40′.
159.—4¾x5; R at (2½, 2¼). Ans. 26°34′.
160.—5¼x2¾; R at (3⅞, 1⅜). Ans. 63°20′.
161.—5x3½; G at (1⅛, 1⅞).
162.—2½x3¾; R at (⅛, 1).
163.—3½x3¾; G at (¾, 2¼).
164.—4x4; R at (⅛, 1½).
165.—5x5; S at (2, 2½).
166.—2¾x2¾; Left end of G. L. at (⅛, 1⅜). Ans. 0.60″.
167.—4¾x4¾; Intersec. bet. G. L. and profile plane at (2⅜, 2⅜). Ans. 1.45″.
168.—4¾x4; R at (1¾, 2¼).
169.—5x7; R at (1½, 2).
170.—2x4¼; U at (⅛, 1¼).
171.—3x4¾; U at (⅛, 1⅜).
172.—5¼x3½; G at (1, 1¾).
173.—3½x5¼; S at (¼, 3).
174.—5⅜x 3½; G at (1, 1¾). Ans. 70°45′ or sup.
175.—2¾x3¾; R at (⅛, 1). Ans. 80°10′ or sup.
176.—3¼x3½; S at (⅛, 2⅜). Ans. 75°15′ or sup.
177.—2¾x2¾; G. L. 1⅜″ up. Ans. 57° or sup.
178.—4¾x4¾; Intersec. bet. G. L. and profile plane at (2⅜, 2⅜). Ans. 32°50′ or sup.
179.—6x2; R at (5⅞, 1⅜). Ans. 46°15′ or sup.
180.—5x2⅜; U at (3¼, 1). Ans. 61°20′ or sup.

181.—4x6; R at (1/8, 4 1/8). Ans. 39°14′ with H, 63°26′ with V.

182.—3 1/4x4 1/2; U at (1/8, 1 1/4). Ans. 73°54′ with H, 33°41′ with V.

183.—3 3/4x3 1/4; K at (1 1/8, 3/4). Ans. 49°45′ with H, 67°45′ with V.

184.—6x10; K at (1, 5). Ans. 65°45′.

185.—5x6; U at (4 7/8, 2 1/2). Ans. 50°46′.

186.—4 1/4x3 1/4; G at (1/8, 1 3/4). Ans. 5°40′.

187.—5x4 1/4; S at (1/4, 2 1/8). Ans. 45°20′.

188.—3x5; S at (1 1/2, 2 1/2).

189.—3x2 1/4; G at (1/8, 1 3/8). Ans. 1.30″ below G. L.

190.—3 3/8x2 1/8; S at (1 3/8, 2). Ans. 1.95″ below G. L.

191.—4 1/4x5 3/8; S at (3/8, 2 5/8).

192.—2 3/4x3 5/8; G at (1/8, 2 3/8).

193.—7 7/8x7 5/8; U at (2 5/8, 3 5/8). Ans. 51°45′ or 61°20′.

194.—6 3/8x8 1/8; S at (3 1/8, 4). Ans. 19°30′ or 30°50′.

195.—6 1/4x5 1/2; K at (1 1/8, 3 7/8). Ans. 0.8″.

196.—9 1/2x10 1/2; S at (2 3/4, 4 3/4). Ans. 0.88″.

197.—6 1/2x9 1/2; S at (2 3/8, 5). Ans. 0.46″.

198.—6x4 1/4; G at (2 5/8, 2 3/8). Ans. 0.60″.

199.—2 3/4x2 3/8; G at (1/8, 1 5/8). Ans. 2.29″.

200.—4 3/4x8; G at (1/8, 2 1/4). Ans. 1.95″.

201.—3 1/2x2 1/2; *d* at (2 1/8, 1/8).

202.—4 3/4x5 1/4; R at (2 1/4, 3 1/4).

203.—6 1/4x5 1/2; R at (2 1/4, 3 3/8).

204.—6 1/4x5 1/2; R at (2 1/4, 3 3/8).

205.—7x5 7/8; *b′* at (4 1/2, 3 3/8).

206.—8 1/8x6 1/2; S at (5 1/2, 3 1/2).

207.—2 3/4x5; K at (1/8, 2).

208.—4 1/4x3 5/8; *n* at (2 1/8, 1 5/8).

209.—4 1/8x5 1/4; *m* at (1/8, 1 5/8).

210.—7 3/4x7 1/2; G. L. 4″ up. Ans. 0.577 ft.

211.—7 3/4x7 1/2; G. L. 4″ up. Ans. 0.27″.

212.—7 3/4x7 1/2; G. L. 4″ up. Ans. 0.92″.

213.—3 1/2x3 1/2; T at (1/8, 1 1/2). Ans. 33°41′.

214.—3x4¼ ; T at (⅛, 1½). Ans. 0.707″.
215.— Ans. 58°49′.
216.— Ans. 47°27′.
217.—5x5¼ ; S at (3½, 2⅜).
218.—2x2¾ ; K at (⅛, 1⅝).
219.—2¾x3⅛ ; R at (⅛, 1⅞).
220.—5x5½ ; *d* at (2⅛, 2⅞). Ans. 55°.
221.—7¼x8½ ; *b′* at (¾, 3¾).
222.—4⅞x4⅛ ; S at (1¾, 2⅛).
223.—2¾x3 ; G at (⅛, 1⅜).
224.—6x6¼ ; S at (2, 1¾). Ans. 1.21″.
225.—8½x7 ; U at (5⅛, 6).
226.—8½x7⅜ ; R at (¾, 3¾).
227.—6¼x8¼ ; R at (4⅝, 3⅝).
228.—5¾x11½ ; U at (⅛, 3¼).
229.—4¼x4¼ ; *b′* at (1⅛, 2⅛).
230.—4⅝x4⅝ ; *a* at (⅛, 1¼).
231.—6¼x4¾ ; *b′* at (1⅛, 2⅝).
232.—7¼x11 ; *b′* at (2⅛, 6¼).
233.—3¾x5 ; S at (1, 2⅛).
234.—4¼x8¾ ; Middle of H proj. at (2⅛, 2⅛).
235.—5¼x7½ ; R at (⅛, 1⅞).
236.—4½x5⅝ ; G at (⅝, 1⅞).
237.—3¼x4¼ ; G at (1⅝, 1⅞).
238.—4¼x6¼ ; S at (2⅛, 3⅝).
239.—7½x9½ ; *c* at (1¼, 3¾).
240.—3x4.
241.—5x5¾.
242.—4x5.
243.—9½x7⅝ ; U at (2¾, 3⅞).
244.—8¼x11⅜ ; R at (1¼, 6).
245.—9x11⅜ ; R at (1½, 6).
246.—6½x9 ; *c′* at (2⅝, 3¼).
247.—5⅜x8⅞ ; *c′* at (1⅛, 5⅜).
248.—6⅞x6¼ ; R at (2⅞, 2¾).
249.—4x2⅜ ; R at (1⅛, 1½).

250.—5½x6¼; G at (⅛, 4½).
251.—16x12; G at (7½, 9½).
252.—7x5¾; G at (⅛, 3⅜).
253.—6½x6⅝; *c′* at (2⅛, 4½).
254.—9x8¼; *c′* at (4⅛, 4⅛).
255.—4½x7¼; *b′* at (¾, 4¾).
256.—4¼x5; *w′* at (2, 3¼). Ans. 0.84″.
257.—7⅝x9⅛; *b′* at (⅛, 5¾). Ans. 1.66″.
258.—3¼x4¼; *b′* at (⅛, 2⅝). Ans. 1.49″.
259.—4¾x8; *b* at (⅞, 2⅜). Put *bc* ‖ to G. L. and 2½″ below it. Ans. 2.04″.
260.—4¾x8; *bc* as in Prob. 259. Ans. 1.63″.
261.—4x6¼; *b′* at (⅝, 4⅛). Ans. (a), 0.09″; (b), 1.95″.
262.—5½x7⅛; *e′* at (1¼, 3½). Ans. (a), 0.75″; (b), 0.99″.
263.—6x6½; *b′* at (1⅝, 3). Ans. Rad. = 1.98″. Center 1.62″ above H.
264.—9¼x10⅝; *b′* at (2, 6½). Ans. 0.45″.
265.—8½x12¾; *b′* at (1, 8½). Put *bc* ‖ to G. L. and 2″ below it. Ans. 0.51″.
266.—A = 59°4′; B = 88°12′; a = 50°2′.
267.—B = 90°; C = 45°44′; c = 45°12′.
268.—A = 80°20′; B = 73°47′; b = 61°47′.
269.—A = 51°58′; C = 83°55′; c = 51°.
270.—a = 68°46′; c = 63°12′; A = 74°.
271.—b = 63°15′; c = 47°42′; C = 55°53′.
272.—a = 38°; b = 42°; B = 58°53′.
273.—b = 48°3′; c = 64°47′; C = 80°20′.
274.—A = 75°54′; B = 49°16′; c = 63°36′.
275.—A = 82°9′; B = 90°; c = 45°12′.
276.—B = 88°12′; C = 55°53′; a = 50°2′.
277.—A = 80°20′; B = 73°47′; c = 48°3′.
278.—A = 82°9′; b = 82°17′; c = 45°12′.
279.—b = 63°15′; c = 47°42′; A = 59°4′.
280.—a = 51°; b = 38°; C = 58°53′.

281.—$b = 61°47'$; $c = 48°3'$; $A = 80°20'$.
282.—$A = 83°55'$; $B = 51°58'$; $C = 58°53'$.
283.—$A = 67°$; $B = 74°$; $C = 82°$.
284.—$A = 55°53'$; $B = 59°4'$; $C = 88°12'$.
285.—$A = 54°8'$; $B = 80°20'$; $C = 73°47'$.
286.—$a = 51°$; $b = 38°$; $c = 42°$.
287.—$a = 48°3'$; $b = 64°47'$; $c = 61°\ 47'$.
288.—$a = 45°12'$; $b = 79°1'$; $c = 82°17'$.
289.—$a = 50°2'$; $b = 63°15'$; $c = 47°42'$.

290.—9¾x7; p at (2⅜, ⅛). Take $pp' = 4¾''$; $pp'' = 5¼''$. Ans. $A' = 33°41'$; $B' = 36°52'$; $C' = 21°48'$; $D' = 47°58'$; $E' = 48°1'$; $F' = 26°21'$; $G' = 36°29'$; $H' = 41°44'$; $I' = 44°50'$; $J' = -18°8'$; $K' = 71°30'$.

291.—8½x6¼; l at (4⅝, 2⅛). Ans. $A' = 33°41'$; $B' = 56°19'$; $C' = 29°1'$; $D' = 29°1'$; $E' = 38°42'$; $F' = 17°55'$; $G' = 20°18'$; $H' = 27°29'$; $I' = 27°50'$; $J' = -4°31'$; $K' = 80°30'$.

292.—6x6¼; l at (2⅛, 2⅛). Ans. $A'' = 45°$; $B'' = 33°41'$; $C'' = 29°1'$; $D'' = 46°41'$; $E'' = 60°59'$; $F'' = 36°2'$; $G'' = 46°41'$; $H'' = 36°2'$; $I'' = 40°37'$; $J'' = -9°30'$; $K'' = 64°30'$.

293.—14x12; w at (4⅛, 4⅛). Take $ym' = 4''$. Ans. $A'' = 45°$; $B'' = 67°30'$; $C'' = 42°44'$; $D'' = 16°5'$; $E'' = 47°5'$; $F'' = 15°35'$; $G'' = 16°5'$; $H'' = 15°33'$; $I'' = 15°35'$; $J'' = -1°5'$; $K'' = 86°$.

294.—12x16; w at (2⅛, 8). Take $ym' = 3''$. Ans $A'' = 33°41'$; $B'' = 69°10'$; $C'' = 32°$; $D'' = 17°35'$; $E'' = 35°30'$; $F'' = 11°30'$; $G'' = 12°$; $H'' = 17°10'$; $I'' = 17°11'$; $J'' = -1°5'$; $K'' = 93°30'$.

295.—11¼x8¼; w at (4¾, ⅛). Take $ym' = 3''$. Ans. $A''' = 38°40'$; $B''' = 50°50'$; $C''' = 32°$; $D''' = 32°15'$; $E''' = 45°35'$; $F''' = 23°10'$; $G''' = 26°50'$; $H''' = 29°10'$; $I''' = 30°$; $J''' = -6°50'$; $K''' = 78°$.

296.—12x16; p at (½, 6½). Take $ym' = 4''$. Ans. $A = 38°40'$; $B = 51°20'$; $C = 32°$; $D = 32°$; $E = 45°42'$;

F = 22°58′; G = 68°11′; H = 29°12′; I = 29°54′; J = —48°25′; K = 78°.

297.—16x12; p at (3¾, ½). Take ym' = 4″. Ans. A′ = 45°; B′ = 38°40′; C′ = 32°; D′ = 41°28′; E′ = 58°; F′ = 33°31′; G′ = 41°28′; H′ = 33°31′; I′ = 36°23′; J′ = —15°5′; K′ = 64°40′.

298.—12x12; l at (3½, 2¾). Take nm'' = 3″. Ans. A′ = 26°34′; B′ = 58°; C′ = 22°59′; D′ = 29°12′; E′ = 30°31′; F′ = 13°43′; G′ = 15°37′; H′ = 28°18′; I′ = 28°30′; J′ = — 3°36′; K′ = 82°45′.

299.—16x12; l at (10, 3¼). Take nm'' = 3″. Ans. A″ = 38°40′; B″ = 32°; C″ = 22°59′; D″ = 51°20′; E″ = 56°29′; F″ = 31°59′; G″ = 44°59′; H″ = 41°28′; I″ = 46°39′; J″ = —23°40′; K″ = 65°40′.

300.—3¼x7½; Center of H proj. at (3⅛, 3⅛).

301.—6x10¼; Center of base of upright cone at (3, 3).

302.—2¼x5½; Center of H proj. at (1⅛, 1⅛).

304.—9½x7¾; e' at (4¼, 3½).

305.—4½x3½; Center of base at (2¼, ⅝).

306.—5x4½; Center of H proj. of base at (2½, ¾).

307.—3x3; R at (2¼, 1¼). Ans. 41°48′.

308.—4x6½; Center of H proj. at (2⅛, 1⅞).

309.—7x5¼; R at (1¼, 2⅝).

310.—2¾x4¼; w' at (1⅛, 2⅛).

311.—6½x4⅜; c' at (1¼, 2⅛).

312.—4¼x4¼; S at (2⅛, 2⅛). Ans. 0.95″.

313.—6¼x7; R at (2, 3½).

314.—2¼x5¼; n' at (1⅛, 2⅛).

315.—8x7; S at (5½, 5).

316.—8¾x7¼; b' at (1¼, 4¾).

317.—1¾x4¼; n' at (⅞, 1⅞).

318.—1¾x4¾; n' at (⅞, 1⅞).

319.—4x10¾; Center of H proj. at (2⅜, 4¾).

320.—4¾x9¾; Intersec. bet. G. L. and V proj. of axis at (2⅞, 6¼).

321.—3½x3¾; K at (1¾, 2). Ans. 30°10′.

322.—3¼x4¾; K at (1¼, 2⅞). Ans. 55°30′.
323.—4¼x5⅛; R at (2½, 2⅛). Ans. 36°20′.
324.—6¾x4; R at (4⅝, 2⅜). Ans. 30°10′.
325.—7x4¼; K at (3½, 2⅝). Ans. 60°.
326.—3¾x3½; *c* at (2, ⅞). Ans. 1.48″ and 0.85″.
327.—4x4; *e′* at (2, 2¾). Ans., H trace, 0.96″, V trace, 0.72″.
328.—5x5; K at (2¾, 2¾). Ans. 147°16′ or sup.
329.—4¼x5; K at (¾, 3). Ans. 52°25′ or sup.
330.—5x8; K at (3⅛, 5⅜). Ans. 78° and 67°45′.
331.—4¾x4⅝; R at (2¼, 2½). Ans. 42°10′ and 49°50′.
332.—7¾x5¾; R at (3½, 2¼). Ans. 18°30′.
333.—5½x3¼; K at (⅝, 1⅜). Ans. 5°5′.
334.—6x5½; *q* at (3⅝; 1⅜). Ans. 2.02″.
335.—4⅝x4; *a* at (2¼, ⅛). Ans. 0.35″.
336.—4½x4; W at (4¼, 2⅜). Ans. 0.05″.
337.—4⅞x3¾; R at (2½, 2⅜). Ans. 0.98″ and 1.47″.
338.—5¾x4¼; K at (2¼, 2⅜). Ans. 46°10′.
339.—4½x5½; K at (1¼, 3). Ans. 18°30′ or sup.
340.—5¾x6¾; R at (3, 2⅞). Ans. 15°45′ or sup.
341.—5x4; K at (⅞, 2). Ans. 2°10′.
342.—5x4; K at (⅞, 2). Ans. 14°55′.
343.—6⅜x4¼; K at (2⅛, 2½). Ans. 14°50′.
344.—3x5⅝; R at (2⅜, 2⅛). Ans. 3.46″ or 1.30″.
345.—5¼x4½; *a* at (⅛, ⅛). Ans. 0.32″ or 1.08″.
346.—4¾x4¼; *o* at (¾, 1⅞). Ans. 0.32″ or 0.76″.
347.—5¼x5¼; *o* at (2⅜, 1⅜). Ans. 0.10″ or 2.18″.
348.—9x7½; R at (5½, 4¾). Ans. 20°45′ or sup.
349.—5x8¼; K at (1⅜, 4⅝). Ans. 2°15′.
350.—8x4¼; R at (4¾, 2⅜). Ans. 14°30′.
351.—7¾x7¼; R at (3⅝, 4⅝). Ans. 5°.
352.—10½x6⅛; *b* at (2⅞, 1⅛). Ans. 20°50′.
353.—11x5½; W at (4⅜, 2¼). Ans. 12°50′.
354.—2¾x3¼; W at (2⅝, 1¾). Ans. 66°.
355.—3¾x4¼; K at (2, 2⅜). Ans. 33°45′ and 77°20′.
356.—9x4¼; R at (6⅞, 2⅜). Ans. 18°30′.

357.—8¼x5¼; K at (3¼, 3⅜). Ans. 32°20′.
358.—3¾x7¾; R at (2¾, 2⅛). Ans. 39°40′.
359.—10⅜x6⅝; W at (2⅞, 3⅞). Ans. 14° and 21°.
360.—13½x6½; W at (6¼, 3½). Ans. 16°50′ and 22°50′.
361.—4¾x7¾; R at (3¾, 2⅜). Ans. 52°.
362.—6⅞x4¼; K at (¾, 2⅜). Ans. 11°10′.
363.—8¼x8¼; R at (6½, 4⅝). Ans. 17°45′.
364.—5¾x8¼; K at (2⅜, 4⅝). Ans. 67°30′.
365.—5¾x4¼; R at (3¾, 2⅛). Ans. 18°50′ and 28°40′.
366.—11½x8¼; *d* at (4½, 3¾). Ans. 14° and 16°.
367.—3¼x4⅛; K at (1⅛, 2⅜). Ans. 14°30′ and 17°
368.—8x6⅜; Vertex at (2½, 2⅜). Ans. 38°45′ and 67°15′.
369.—8¾x4½; K at (7, 2½). Ans. 16° or 54°50′.
370.—9x5¼; K at (3, 3⅜).
371.—4¾x6½; Intersection of G. L. with V proj. of axis at (1⅞, 4⅛). Ans. 20°47′.
372.—9¾x7; Intersection of G. L. with V proj. of axis at (7¾, 4⅝). Ans. 15°.
373.—10¾x7½; Intersection of G. L. and V proj. of axis at (9½, 4¾). Ans. 17°7′.
374.—5⅜x6¼; Intersection of G. L. with V proj. of axis at (2⅛, 4⅝). Ans 11°15′.
375.—13¾x7½; Intersection of G. L. with V proj. of axis at (8⅝, 5¾). Ans. 12°28′ or 3°15′.
376.—16x12; Intersection of G. L. with V proj. of axis at (10¼, 8¾). Ans., (a) 2.45″ or 10.07″, (b) 11°.
377.—2½x4¾; *b′* at (⅜, 2½).
378.—7x6¼; K at (⅝, 2¼).
379.—7¾x6½; *b′* at (1⅞, 3½).
380.—10¼x9½; S at (2⅛, 4⅛). Ans. 0.96″.
381.—10¾x6¾; S at (3⅞, 3⅝). Ans. 3.79″.
382.—4x5; *w′* at (2, 2⅜). Ans. 59°50′.
383.—4¼x7¾; *e′* at (⅛, 4⅛). Ans. 79°15′.
384.—4¼x8⅝; *w* at (2⅛, 4⅝). Ans. 11°25′.

385.—7½x10; Intersec. bet. G. L. and V proj. of axis at (4⅞, 6½). Ans. 42°59′.

386.—5x8¼; Intersec. bet. G. L. and V proj. of axis at (2⅛, 5⅜). Ans. 45°43′.

387.—6x9¾; Intersec. bet. G. L. and V proj. of axis at (3¾, 6⅛). Ans. 46°48′.

388.—8x7; *e′* at (4, 3⅛).

389.—8⅛x5½; K at (2⅝, 3⅜). Ans. 5°.

390.—7x7½; R at (3¾, 4¾).

391.—4½x6¾; T at (1⅝, 3⅞). Ans. 76°50′.

392.—2¼x4¼; *d* at (1⅛, 1⅛). Ans. 9°55′.

393.—3½x7; T at (1¾, 4). Ans. 83°30′.

394.—7½x9⅜; W at (1¼, 3¼). Ans. 3.02″.

395.—4⅝x2⅝; K at (1⅜, 1⅜). Ans. 3.18″.

396.—3¾x3¼; R at (2⅛, 1⅝). Ans. 2.01″.

397.—4¼x6¾; *a′* at (2⅝, 2⅝). Ans. 2.43″.

398.—4⅝x6¾; *a′* at (1⅝, 3⅛). Ans. 0.55″.

399.—6¼x9½; *a′* at (1⅝, 5). Ans. 1.50″.

400.—6x6¼; *a′* at (4⅜, 2⅝). Ans. 1.47″ and 2.83″.

401.—12½x8; *a′* at (2¼, 5). Ans. 10°40′.

402.—6½x7; *a′* at (2⅜, 3⅞). Ans. 23°15′.

403.—6x7; *a′* at (3⅝, 3⅞). Ans. 21°30′.

404.—8x10; *a′* at (4½, 3).

405.—9¼x7; *a′* at (2¾, 3¼). Ans. 7°20′ and 67°30′.

406.—13½x10; *c′* at (4, 4¼). Ans. 11°15′ and 13°50′

407.—11¼x6½; R at (8⅜, 3½). Ans. 4°45′ and 10°15′.

408.—6x5¾; *a′* at (2⅛, 3¼). Ans. 15°10′ and 106°15′.

409.—13¼x5; G at (1¾, 3¾). Ans. 13°45′ and 36°15′.

410.—11¾x11½; *b′* at (4⅛, 8). Ans. 2°50′ and 11°15′.

411.—7¼x7½; *a′* at (2¾, 4). Ans. 70°5′.

412.—9¼x9¼; *a′* at (2¼, 3⅞). Ans. 53°15′.

413.—16x12; *b′* at (7⅛, 6).

414.—15x12; *b′* at (4½, 6).

415.—16x12; S at (7¼, 6¼).

416.—16x12; *b′* at (7¼, 8¼).

417.—4¾x7½; *a′* at (2⅜, 3⅞).

418.—5⅛x5; *n′* at (1¼, 2⅛).

419.—3⅝x4¼; *o* at (⅛, 2⅜).

420.—4½x3¼; R at (3¾, 2⅛).

421.—3½x4¾; K at (⅝, 2⅝).

422.—5x4¼; K at (¾, 2⅜).

423.—2½x4¼; K at (¾, 2⅜).

424.—4¾x8¼; K at (1⅜, 4⅝).

425.—6¼x7¼; *e′* at (3⅛, 4⅛).

426.—3⅝x5¾; *o* at (2⅜, 1⅝).

427.—15x11½; Intersec. bet. G. L. and V proj. of axis at (2⅛, 5¾). Put lower left hand corner of development at (2⅛, 8¼).

428.—15½x6½; Intersec. bet. G. L. and V proj. of axis at (2⅝, 3¾). Put lower left hand corner of pattern at (2⅛, 8¼).

429.—10x9½; Put construction work on left, and pattern on right.

430.—11½x11⅞; R at (2, 6⅜).

431.—11x15; S at (2¾, 6¾). Middle of developed right section at (4½, 13). Ans. Length of pattern = 8.69″. Length of longest element = 2.37″.

432.—11x10; K at (1¾, 6½). Middle of developed right section at (10¾, 5½). Ans. Length of pattern = 8.64″.

433.—15¾x8⅜; Center of base at (1¼, 1½).

434.—7½x14½; Vertex at (5⅛, 9½). Ans. (b) maj. = 3.48″, min. = 2.86″, (c) 161°.

435.—4½x6½; Intersec. bet. G. L. and V proj. of axis at (2¼, 2¼).

436.—12x7¾; Intersec. bet. G. L. and V proj. of axis at (2⅝, 5⅛). In pattern, put vertex at (8⅜, 7).

437.—10¼x6¾; Intersec. bet. G. L. and V proj. of axis at (1⅞, 3⅝). In development, put vertex at (6⅝, 6⅛).

438.—14½x10¾; R at (⅛, 4¾). Vertex in development at (10¼, 10⅝).

439.—13x8½; R at (⅛, 4⅛).

440.—$14\frac{1}{2}$x$8\frac{3}{8}$; T at ($\frac{1}{8}$, $4\frac{5}{8}$). Vertex for pattern at ($10\frac{1}{2}$, $6\frac{5}{8}$).

441.—13x$7\frac{1}{4}$; R at ($5\frac{3}{8}$, $4\frac{1}{8}$).

442.—$8\frac{1}{2}$x$13\frac{1}{2}$; n' at ($3\frac{3}{4}$, $6\frac{3}{4}$).

443.—16x11; Center of developed helix at ($4\frac{1}{4}$, 8).

444.—$4\frac{1}{4}$x$8\frac{3}{4}$; n' at ($1\frac{1}{2}$, $5\frac{5}{8}$).

445.—$6\frac{1}{2}$x$6\frac{1}{2}$; Center of developed helix at (4, 4).

446.—$7\frac{1}{2}$x$6\frac{3}{4}$; n' at ($1\frac{5}{8}$, $3\frac{5}{8}$).

447.—$6\frac{3}{4}$x$7\frac{7}{8}$; a' at ($2\frac{3}{8}$, $4\frac{3}{4}$).

448.—$8\frac{1}{4}$x11; a' at ($2\frac{3}{8}$, $3\frac{7}{8}$).

449.—$6\frac{1}{2}$x5; R at ($\frac{1}{4}$, $2\frac{1}{8}$).

450.—$6\frac{3}{4}$x$6\frac{1}{2}$; R at ($\frac{1}{8}$, 3).

451.—10x12; R at ($4\frac{3}{8}$, $8\frac{5}{8}$).

452.—$8\frac{1}{4}$x$6\frac{3}{4}$; n' at ($2\frac{1}{8}$, $3\frac{5}{8}$).

453.—Let the floor be taken coincident with the H plane of proj.

454.—$10\frac{1}{2}$x16; n' at ($6\frac{3}{4}$, $6\frac{1}{4}$).

455.—11x12; Intersec. bet. G. L. and V proj. of axis of cone at ($6\frac{3}{4}$, $4\frac{3}{8}$). Ans. $\angle$ at vertex of developed cone $= 186°35'$.

456.—14x$10\frac{1}{4}$; Intersec. of tank axis and G. L. at (2, $4\frac{1}{4}$); Lower left corner of developed tank at ($4\frac{3}{8}$, 0); Lower left corner of developed pipe at (6, 5); Vertex of developed cone at ($14\frac{1}{8}$, $7\frac{3}{8}$). Ans. Length tank pattern $= 12.57''$; Length pipe pattern $= 4.71''$; Angle at vertex of developed cone $= 254°25'$.

457.—$13\frac{1}{2}$x9; K at ($6\frac{1}{2}$, $4\frac{5}{8}$).

458.—10x16; b' at ($4\frac{1}{8}$, $9\frac{3}{8}$).

459.—15x$10\frac{1}{4}$; Lower left hand corner of Fig. 17 at ($1\frac{5}{8}$, $1\frac{5}{8}$).

460.—$13\frac{1}{4}$x$10\frac{1}{2}$; Put G. L. 4" up.

461.—$10\frac{3}{4}$x$10\frac{3}{4}$; Intersec. bet. G. L. and V proj. of axis at ($5\frac{3}{8}$, $5\frac{3}{8}$).

462.—$7\frac{1}{4}$x15; d' at ($5\frac{1}{8}$, $8\frac{1}{4}$).

463.—$9\frac{1}{4}$x11; Intersec. bet. G. L. and V proj. of axis of 1st cone at ($4\frac{1}{4}$, $5\frac{1}{2}$).

464.—11x16; c' at ($5\frac{1}{4}$, $5\frac{1}{2}$). Middle of developed cylindrical base at (11, 11). Vertex of developed cone at (0, 16).

465.—$10\frac{1}{2}$x$9\frac{1}{2}$. Intersec. bet. G. L. and V proj. of axis at ($8\frac{1}{2}$, 4). Vertex of developed cone at ($\frac{1}{8}$, $4\frac{3}{4}$). Ans. $\angle$ at vertex of developed cone $= 109°9'$.

466.—14x10; n' at ($6\frac{1}{4}$, 7).

467.—6x6; c' at ($3\frac{1}{8}$, $3\frac{5}{8}$).

468.—8x$9\frac{3}{4}$; K at ($2\frac{1}{4}$, $5\frac{1}{8}$).

469.—14x$8\frac{1}{4}$; K at ($2\frac{1}{4}$, $4\frac{5}{8}$).

470.—$5\frac{1}{2}$x$8\frac{1}{2}$; a' at ($2\frac{3}{8}$, $3\frac{7}{8}$).

471.—$5\frac{5}{8}$x7; a' at ($2\frac{3}{8}$, $3\frac{7}{8}$).

472.—5x$5\frac{1}{4}$; a' at ($1\frac{5}{8}$, $3\frac{1}{8}$).

473.—$6\frac{3}{4}$x$9\frac{1}{2}$; H proj. of axis of torus at ($3\frac{3}{8}$, $3\frac{3}{8}$).

474.—$4\frac{1}{4}$x$7\frac{1}{4}$; a' at ($1\frac{5}{8}$, $3\frac{5}{8}$).

475.—$3\frac{1}{4}$x$5\frac{1}{2}$; n' at ($2\frac{1}{8}$, $2\frac{3}{8}$).

476.—$5\frac{1}{2}$x$6\frac{3}{4}$; b' at ($2\frac{3}{8}$, $3\frac{3}{8}$).

477.—$11\frac{1}{4}$x$4\frac{3}{4}$; b' at ($4\frac{1}{4}$, $2\frac{5}{8}$).

478.—$5\frac{3}{4}$x$8\frac{1}{4}$; S at ($1\frac{1}{4}$, $4\frac{3}{8}$).

481.—$5\frac{1}{4}$x$11\frac{1}{2}$; Intersec. bet. G. L. and V proj. of axis of helix at ($2\frac{5}{8}$, $5\frac{1}{2}$).

482.—$4\frac{1}{4}$x$5\frac{3}{4}$; b' at ($\frac{7}{8}$, $2\frac{3}{4}$).